St. Joseph's College of Arts and Science, Cuddalore.

Question Bank

PG Research Department of Mathematics

CLASS: I B.SC (CS)

SUBJECT NAME: ALLIED MATHEMATICS II

SUBJECT CODE: AMCS202S

STAFF NAME: Mrs.D.RAMYA

SEMESTER : II

Unit-1

LAPALACE TRANSFORMS

Two Marks

1. Define Laplace transforms.
2. State Linearity property.
3. State change of scale of Laplace transform.
4. Find L$(e^t + \frac{1}{e^t})^2$.
5. Find the Laplace transforms of 3cos 4t.
6. Find the Laplace transforms of 4 sin 3t.
7. Find the Laplace transforms of $e^t + sint$.
8. Find the Laplace transforms of 2^{at}.
9. Find the Laplace transforms of $cos^2 t$.
10. Find the Laplace transforms of $sin^2 t$.

11. Find the Laplace transforms of $sin^2 2t$.

12. Find the Laplace transforms of $cos^2 2t$.

13. Find the value of L(sin 3t cost).

14. Find the L.T's of cos 4t cos 2t.

15. Find the L.T's of cos 6t cos 2t.

16. Find the L.T's of cos 3t cos t.

17. Find the L.T's of cos t cos 2t.

18. Find the L.T's of sin 2t sin t.

19. Find the L.T's of sin 3t sin t.

20. Evaluate L $[(t + 1)^2]$.

21. Define Inverse Laplace transform.

22. Find the inverse L.T's of $\frac{s}{(s+2)^2}$.

23. Find the inverse L.T's of $\frac{s}{(s-4)^2}$.

24. Find the inverse L.T's of $\frac{s+2}{(s-2)^7}$.

25. Find the inverse L.T's of $\frac{s^2}{(s-1)^3}$.

26. Find the inverse L.T's of $\frac{s+1}{s^2+2s+2}$.

27. Find the inverse L.T's of $\frac{s-3}{s^2-6s+13}$.

28. Find the inverse L.T's of $\frac{1}{s^2(s^2+4)}$.

29. Evaluate $L^{-1}\frac{s^3}{s^4-a^4}$.

30. Show that $L^{-1}[\log(\frac{s+a}{s+b})] = -\frac{e^{-at}-e^{-bt}}{t}$

31. Show that $L^{-1}[\log\frac{s+1}{s-1}] = 2\frac{\sinh t}{t}$.

32. Show that $L^{-1}\left[tan^{-1}\frac{a}{s}\right] = \frac{\sin at}{t}$.

33. Find the Laplace transforms of cos (at +b).

34. Find the Laplace transforms of sin (2t +3).

35. Find the Laplace transforms of 3 cosh 2t.

36. Find the Laplace transforms of cosh(at+b).

37. Find the Laplace transforms of 3 sinh 2t.

38. Find the Laplace transforms of sinh(2t+3).

39. Find the L.T's of $2e^{3t} + 3e^{-3t}$.

40. Find the L.T's of $4e^{3t} - 2e^{-3t}$.

41. Find the L.T's of $(e^{t} + e^{-t})^2$.

42. Find the L.T's of 4 sin t cos t.

43. Find the L.T's of $cos^2 t - sin^2 t$.

44. Find the L.T's of $e^{2t} + 3cos^2 t$.

45. Find the L.T's of $e^{3t} + 5sin^2 3t$.

46. Prove that $L^{-1}F(s+a) = e^{-at}L^{-1}F(s)$.

47. Prove that $L^{-1}\left[\frac{F(s)}{s}\right] = \int_0^t L^{-1}[F(s)]dt$.

48. Prove that $L^{-1}[sF(s)] = \frac{d}{dt}L^{-1}F(s)$

49. Prove that $L^{-1}[F(s)] = -\frac{1}{t}L^{-1}F\left[\frac{d}{ds}F(s)\right]$

50. Prove that $L^{-1}[F(s)] = tL^{-1}\int_s^\infty [F(s)]ds$.

51. Find the inverse L.T's of $\frac{s^2}{(s-4)^4}$

52. Find the inverse L.T's of $\frac{s^3}{(s-4)^4}$

53. Find the inverse L.T's of $\frac{s}{(2s-3)^3}$.

54. Find the inverse L.T's of $\frac{s}{(s+1)^2+2^2}$.

55. Find the inverse L.T's of $\frac{s}{(s+2)^2+4}$.

56. Find the inverse L.T's of $\frac{s}{(s-a)^2+b^2}$.

57. Find the inverse L.T's of $\frac{s}{(s-b)^2+a^2}$.

58. Evaluate $L[(t+1)^2]$.

59. Show that $L^{-1}[\log\frac{s}{(s^2+4^2)}] = \frac{1}{t}[4 \cos 2t - 1]$.

60. Find the inverse L.T's of $\frac{s-2}{s^2+2s+2}$.

Five Marks

1. Find the Laplace transforms of sin(2t+3)Find the Laplace transforms of f(t) if $\begin{matrix} f(t)=e^t, & 0\le t\le 4, \\ f(t)=0, & 4<t<\infty \end{matrix}$.
2. Prove that $L[e^{-at}f(t)] = F(s+a)$.
3. Prove that $L[e^{-at}f(t)] = F(s+a)$.
4. Prove that $L\left[\frac{f(t)}{t}\right] = \int_s^\infty F(s)ds$.
5. Prove that $L[f'(t)] = F(s) - f(0)$.
6. Prove that $L\int_0^t F(t)dt = \left[\frac{f(s)}{s}\right]$.
7. Evaluate L $(e^{-3t} sint \cos t)$

8. Evaluate L($e^{3t}(cos^2 t - sin^2 t)$).

9. Evaluate L($e^{-5t} cos^2 t$).

10. Eva^{lua}te L(cosh 3t+sinh 3t).

11. Evaluate L ($e^t\, cos^3 t$).

12. Evaluate L ($t^2 e^{3t}$ sinh t).

13. Find the L.T's of t cos at.

14. Find the L.T's of t^2 cos at.

15. Find the L.T's of t sin at.

16. Find the L.T's of t^2 sin at.

17. Find the L.T's of t sin 3t-cos 2t.

18. Find the L.T's of t cos^2 at.

19. Find the L.T's of t sin^2 at.

20. Show that L(cos at-1/2 at sin at)=$s^3/(s^2+a^2)^2$.

21. Show that L[$1/2(\sin at - at \cos at)$]=$a^3/(s^2+a^2)^2$

22. Prove that the linear property of inverse Laplace transform.

23. Find the inverse L.T's of $\frac{s-3}{s^2+4s+13}$.

24. Find the inverse L.T.'s of $\frac{s-3}{(s-3)^2+4}$.

25. Find the inverse L.T.'s of $\frac{s+1}{s^2+2s+2}$.

26. Find the inverse L.T.'s of $\frac{s}{s^2+2s+10}$.

27. Show that $L^{-1}[\log\frac{s^2+a^2}{s^2+b^2}] = 2\frac{\cos bt-\cos at}{t}$.

28. Show that $L^{-1}[\log\frac{s+a}{s+b}] = \frac{e^{-at}-e^{-bt}}{t}$.

29. Show that $L^{-1}[\log\frac{s+1}{s-1}]$ =2sinh t/t.

30. Show that $L^{-1}[\log\frac{s+a}{s+b}]$ =-1/tL^{-1}[$\frac{d}{ds}\log\frac{s+a}{s+b}$].

Ten Marks

1.Find the Laplace's transform of the following functions. (i) cos^4t (ii) te^{3t}cos 4t.

2. Find the inverse Laplace transform of the following functions. (i) $\frac{2(s+1)}{[(s+1)^2+1]^2}$ (ii) $\frac{1}{s(s^2+a^2)}$.

3. Find the centroid of the area enclosed by one arch of the cycloid x=a($\theta - sin\theta$),y=a(1-cosθ).

4. Find $L^{-1}\left(\frac{1}{s(s+1)(s+2)}\right)$

5. Find $L^{-1}\left(\frac{1+2s}{(s+2)^2(s-1)^2}\right)$

6. . Find $L^{-1}\left(\frac{1}{(s+1)(s^2+2s+2)}\right)$

7. Solve by using Laplace transforms y''+y'-6y=0 given that y=5,y'=0 when t=0.

8. Solve by using Laplace transforms y''+4y'+4y=e^{-x}

Given that y(0)=5,y'(0)=0.

9. Find $L^{-1}\left[\frac{s-3}{s^2+4s+13}\right]$

10. Find f(t) if Lf(t)=$\frac{1}{(s+1)(s^2+2s+2)}$.

11. Solve by Laplace transforms method y”+4y’+4y=e^{-x} given that y(0)=y’(0)=0.

12. Find L[$te^{-1}sint$]

13. Using Laplace’s transforms solve y’’+4y’+8y=cos2x given that y=2 and y’=1 at x=0.

14. Find $L^{-1}\left[\frac{1}{s^2(s^2+1)(s^2+9)}\right]$.

15. Find $L^{-1}\left[\frac{4s^2-3s+5}{(s+1)(s-1)(s-2)}\right]$.

16. Solve by using Laplace transforms y”-3y’+2y=4 given that y=2, y’=3 when x=0.

Unit-2

LAPLACE TRANSFORM(CONTD)

Two Marks

1. Solve $y''+y'-6y=0$ given that $y=0,\quad y'=0$ when $t=0$.
2. Solve $\frac{d^2y}{dt^2}+\frac{dy}{dx}-6y=0$ if $y=0,\quad y'=2$ when $t=0$.
3. Solve $y''+3y'+2y=0$ given that $y=1,\quad y'=2$ when $x=0$.

Five Marks

1. Using Laplace transforms, solve $\frac{d^2y}{dt^2}+6\frac{dy}{dt}+5y=e^{-2t}$, given that y=0, $\frac{dy}{dt}=1$, when t=0.

2. Using Laplace transforms, solve $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{-x}$, given that y=0,$\frac{dy}{dx} = 1$ when x=0.

3. Solve $\frac{d^2y}{dx^2} + 4y = 5e^{-x}$, using Laplace transform, given that y(0)=2, y'(0)=3.

4. Solve by L.T. method$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 13y = 2e^{-x}$, given that y(0)=0, y'(0)=-1.

5. Using Laplace transforms, solve $\frac{d^2y}{dx^2} - y = x^2 + x$, given that y(0)=y'(0)=0.

6. Using Laplace transforms, solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dt} - y = 3sint$, given that y(0)=2 and y'(0)=0

7. Using Laplace transforms solve the differential equation $\frac{d^2y}{dt^2}$ - 4$\frac{dy}{dt}$-5y=te^t given y(0),y'(0)=0

8. Solve y"-3y'+2y=sin t given y(0)=0,y'(0)=-1 using Laplace transforms.

9. Using Laplace transforms solve y"-3y'+2y=4 given that y=2, y'=3 when x=0.

10. Solve $x^3y''' + 3x^2y'' + xy' + y = x^2 + logx$.

11. Solve y''+4y=tan2x by the method of variation of parameters.

12. Solve $\frac{d^2y}{dt^2}$ +2$\frac{dy}{dt}$-3y=sint given that y=$\frac{dy}{dt} = 0$ when t=0.

13. Solve $\frac{d^2y}{dt^2}$ +4$\frac{dy}{dt}$-5y=5 given that y=0$\frac{dy}{dt}$=2 when t=0.

14. Solve $\frac{d^2y}{dt^2}$ +4$\frac{dy}{dt} + 13y$=2e^{-x} , by using Laplace transforms given that y(0)=0,y'(0)=-1.

15. Solve y''+4y=tan 2x, using variation of parameters.

16. Solve $x^2(^{d^2y}/_{dx^2})$+7x($^{dy}/_{dx}$)+8y=logx.

Ten Marks

1. . Solve$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-x}$, using Laplace transform, given that y(0)=0, y'(0)=1.
2. Using Laplace transforms, solve $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 5y = e^{-2t}$, given that y=0,$\frac{dy}{dx} = 1$ when t=0.
3. Using Laplace transforms, solve $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{-x}$, given that y=0,$\frac{dy}{dx} = 1$ when x=0.

4. Solve $\frac{d^2y}{dx^2} + 4y = 5e^{-x}$, using Laplace transforms, given that $y(0) = 2, y'(0) = 3$.
5. Solve by L.T. method $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 13y = 2e^{-x}$, given that $y(0) = 0, y'(0) = -1$.
6. Using Laplace transforms, Solve $\frac{d^2y}{dx^2} - y = x^2 + x$, given that $y(0) = y'(0) = 0$.
7. Using Laplace transforms, solve $(D^2 + 2D - 3)$y= sin t.
8. Using Laplace transforms,

   Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 3\sin t$, given that $y(0) = 2$ and $y'(0) = 0$

Unit-3

VECTOR DIFFERENTIATION

Two Marks

1. Define Scalar point functions.
2. Define Vector Point functions.
3. State angle between two surfaces.
4. State unit vector normal to the surface.
5. Find the directional derivative of the function $x^2 + y^2 + z^2$ at (3,6,9) in the direction whose d.c.'s are 1/3, 2/3, 2/3.
6. Find the unit vector normal to the surface $x^2 + y^2 + 2z^2 = 4$.

7. Find $\nabla\phi$ if

(i) $\phi = x^2 y^3 z^2$

(ii) $\phi = xyz - x^2$

8. Find $\nabla\phi$ in the following cases at the points specified:

(i) $\phi(x, y, z) = 2xy - y^2$ at (1, 3, 2)

(ii) $\phi(x, y, z) = x = xy^2 + yz^2$ at (1, 0, 0)

(iii) $\phi(x, y, z) = y^2(x - z)$ at (1, 1, 2)

9. Define solenoidal Vector.

10. Define Irrotational Vector.

11. Show that the Vector $\vec{A} = x^2 z^2 \vec{i} + xyz^2 \vec{j} - xz^3 \vec{k}$ solenoid.

12.Determine the constant a so that the vector $\vec{F} = (x + 2y)\vec{i} + (y - 2z)\vec{j} + (x + az)\vec{k}.$

13.Show that $\vec{F} = yz\vec{i} + zx\vec{j} + xy\vec{k}$ is irrotational.

14. If $\vec{A} = axy\vec{i} + (x^2 + 2yz)\vec{j} + y^2\vec{k}$ is irrotational, find the value of a.

15. If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, show that $\nabla.\vec{r} = 3$.

16. Show that $x^2\vec{i} + 2xy\vec{j} - 4xz\vec{k}$, is solenoidal.

17. Find the values of m if the following vectors are solenoidal:

(i) $\vec{r} = (x + 2y)\vec{i} + (my + 4z)\vec{j} + (5z + 6x)\vec{k},$

(ii) $\vec{r} = (2x + y)\vec{i} + (4x - 11y + 3z)\vec{j} + (3x + mz)\vec{k},$

(iii) $\vec{r} = (x + 3y)\vec{i} + (y - 2z)\vec{j} + (x + mz)\vec{k}.$

18. Show that $\vec{A} = (4xy - z^3)\vec{i} + 2x^2\vec{j} - 3xz^2\vec{k}$ is irrotatioanl.

19. Define Laplacian operator.

20. If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $\vec{r} = r\hat{r}$, show that (i) $\nabla.[f(r)\vec{r}] = rf'(r) + 3f(r)$

(ii) $\nabla \times [f(r)\vec{r}] = \vec{0}$

21. If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, find n so that $r^n\vec{r}$ is solenoidal.

22.Define Scalar Potential.

23. Define scalar point function with example.

24. Define vector point function with example.

25. Define directional derivative of a scalar point function.

26. Define gradient of a scalar point function.

27. Define angle between two surfaces.

28. Find the equation of the tangent plane to the ellipsoid $x^2 + 2y^2 + 3z^2 = 6$ at (1,1,1).

29. If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, show that $\nabla.\vec{r}$=3.

30. Find the divergence of the vector point function $\vec{A} = x^2yz\vec{i} + xyz^2\vec{j} + \vec{k}$.

31. Show that $x^2\vec{i} + 2xy\vec{j} - 4xz\vec{k}$ is solenoidal.

32. Define scalar potential.

33. Define line integrals.

34. Find $L^{-1}\left[\frac{3s+2}{s}\right]$.

35. Find L[sin3t cost].

36. Find L[$sin^2 t$].

37. Find L[3cos4t].

38. Solve p^2-3p+2=0.

39. Solve (D^2-9)y=0.

40. Solve $(D^2 + 5D + 4)Y = 0$.

41. Find $\vec{F}= x^2\vec{i}+y^2\vec{j}+z^2\vec{k}$ then find div $\vec{F}$.

42. Find the unit normal to the surface x^3-xyz+y^3=1 at(1,1,1).

Five Marks

1. Prove that the directional derivative of ϕ=x^3y^2z at (1,2,3) is a maximum along the direction $9\vec{i}+3\vec{j}+\vec{k}$. Find this maximum directional derivative.
2. Find the directional derivative of the following scalar point functions at the given points in the given directions.

Si. No.	Function ϕ	Point	Direction
(i)	xy+yz+zx	(1, 1, 3)	$\vec{i}+2\vec{j}+2\vec{k}$

(ii)	Xyz	(2, 1, 1)	$\vec{j}+\vec{k}$
(iii)	$x^2yz+4xz^2$	(1, -2, -1)	$\vec{2i}-\vec{j}-\vec{2k}$
(iv)	2xy+5yz+zx	(1, 2, 3)	$\vec{3i}-\vec{5j}+\vec{4k}$

3. Find the angle between the surfaces $x^2+y^2+2z^2=4$, $z=x^2+y^2-3$ at the point (2, -1, 2).

4. Find the equation of the tangent plane to the ellipsoid $x^2+2y^2+3z^2=6$ at (1, 1, 1).

5. Find the directional derivatives of ϕ at the given point in the directional of the given vector.

Si. No	Functions ϕ	Point	Vector
(i)	$2x^2+3y^2+z^2$	(2, 1, 3)	$\vec{i}-\vec{2k}$
(ii)	xy^2	(1, 1, 0)	$\vec{i}+\vec{j}+\vec{k}$
(iii)	Yz^2	(0, 1, 1)	$\vec{i}+\vec{j}+\vec{k}$
(iv)	$3xy^2-x^2yz$	(1, 2, 3)	$\vec{i}-\vec{2j}+\vec{2k}$
(v)	$xyz-xy^2z^3$	(1, 2, -1)	$\vec{i}-\vec{j}-\vec{3k}$
(vi)	$X^3+y^3+z^3$	(1, -1, 2)	$\vec{i}+\vec{2j}+\vec{k}$
(vii)	$4xz^2+2xyz$	(1, 2, 3)	$\vec{2i}+\vec{j}-\vec{k}$
(viii)	x^2y^2z	(2, 1, 4)	$\vec{i}+\vec{2j}+\vec{2k}$
(ix)	$x^2yz+4xz^2+xyz$	(1, 2, 3)	$\vec{2i}+\vec{j}-\vec{k}$

6. Find the unit vectors $\hat{n}$ normal to the following surfaces at the specified points.

(i) $x^2+2y^2+z^2=7$ at (1, -1, 2)

(ii) $x^2+y^2-z^2=1$ at (1, 1, 1)

(iii) $x^2+3y^2+2z^2=6$ at (2, 0, 1)

(iv) $x^2+y^2-z=1$ at(1, 1, 1)

(v) $x^3-xyz+y^3=1$ at (1, 1, 1)

7. Find the maximum value of directional derivative of the function $\phi = 2x + 3y^2 + 5z^2$ at the point (1, 1, -4).
8. Find the equation of the tangent plane to the surface $x^2+2y^2+3z^2=6$ at (1, -1, 1)
9. Find the angle between the normals to the surface $xy-z^2=0$ at the points (1, 4, 2) and (-3, -3, 3).
10. Prove that curl $(\vec{r} \times \vec{a}) = -2\vec{a}$, where $\vec{a}$ is a constant vector.

11. Find Curl $\vec{F}$ in the following cases:

(i) $\vec{F} = x^2 y\vec{i} + y^2 z\vec{j} + z^2 x\vec{k}$

(ii) $\vec{F} = xyz\vec{i} + xyz^2\vec{j} + x^2 yz\vec{k}$

12. Find the value of the constants a, b, c so that

$\vec{F} = (x+2y+az)\vec{i} + (bx-3y-z)\vec{j} + (4x+cy+2z)\vec{k}$

13. If $\vec{F} = xyz\vec{i} + xyz^2\vec{j} + x^2 yz\vec{k}$, then find div curl $\vec{F}$.

14. If $\vec{F} = x^2 y\vec{i} + y^2 z\vec{j} + z^2 x\vec{k}$, then find div curl $\vec{F}$.

15. Find $\nabla \times \vec{F}$ if $\vec{F} = xz^3\vec{i} - 2x^2 yz\vec{j} + 2yz^4\vec{k}$.

16. Find the curl of the following functions at the specified points.

(i) $x^2 z\vec{i} - 2y^3 z^2\vec{j} + xy^2 z\vec{k}$ at (1, -1, 1)

(ii) $xyz\vec{i} + 3x^2 yz\vec{j} + (xz^2 - y^2 z)\vec{k}$ at (1, 2, -1)

17. Find a if $\vec{A}=(4xy-z^3)\vec{i}+ax^2\vec{j}-3xz^2\vec{k}$ is irrotational.

18. If $\nabla.[f(r)\vec{r}]=0$ then show that $f(r)=\frac{C}{r^3}$.

19. Show that $\nabla.\frac{\vec{r}}{r^3}=0.$

20. Show that $\nabla^2 r^n = n(n+1)r^{n-2}$.

21. Show that $(\nabla\phi)\times(\nabla\psi)$ is solenoidal.

22. Show that $\nabla\times(\phi\nabla\phi)=\vec{0}$.

23. Evaluate $\int_c 3x(x+2y)dx+(3x^2-y^3)dy,$ where c is the line joining (0, 0) and (1, 2).

24. If $\vec{F}=3xy\vec{i}-y^3\vec{j}$, compute $\int_c \vec{F}.d\vec{r}$ along $y=2x^2$ from (0, 0) to (1, 2)

25. If $\vec{F}=xz\vec{i}+yz\,\vec{j}+z^2\vec{k}$, evaluate $\int_c \vec{F}.d\vec{r}$ from the point (0, 0, 0) to (1, 1, 1) where c is given by $x=t\ ,y=t^2,z=t^3.$

26. Evaluate the integral where c is the circle $x^2+y^2+z^2=a^2, z=0.$

27. Find the directional derivative of the function $x^2+y^2+z^2$ at (3,6,9) in the direction whose d.c.'s are $\frac{1}{3},\frac{2}{3},\frac{2}{3}$.

28. Prove that the directional derivative of $\emptyset = x^3y^2z$ at (1,2,3) is a maximum along the direction $9\vec{i}+3\vec{j}+\vec{k}$. Find this maximum directional derivative.

29. Find ∅(x,y,z) given that ∅(1,1,1)=3 and $\nabla\emptyset=(y+y^2+z^2)\vec{i}+(x+z+2xy)\vec{j}+(y+2zx)\vec{k}$.

30. Prove that curl $(\vec{r} \times \vec{a})= -2\vec{a}$, where $\vec{a}$ is a constant vector.

31. Prove that $\vec{A}=(2x+yz)\vec{i}+(4y+xz)\vec{j}-(6z-xy)\vec{k}$.

Ten Marks

1. Find $\phi(x, y, z)$ given that $\phi(1, 1,1)=3$ and $\nabla\phi = (y + y^2 + z^2)\vec{i} + (x + z + 2xy)\vec{j} + (y + 2zx)\vec{k}.$
2. Find the angle between the surfaces $x^2+yz=2$ and $x+2y-z=2$ at (1, 1, 1).
3. Find the angle between the surfaces $x^2+y^2=4-5z$ and $x^2+y^2+3z^2=104$ at (5, 2, -5).
4. Find ϕ if $\nabla\phi$ is
   (i) $(6xy+z^3)\vec{i} +(3x^2-z)\vec{j}+(3xz^2-y)\vec{k}$
   (ii) $(2yz+1)\vec{i} +x^2z\vec{j}+x^2y\vec{k}$
   (iii) $(y+\sin z)\vec{i} +x\vec{j}+x \cos z\vec{k}$
   (iv) $(y^2\cos x+z^3)\vec{i} +(2y \sin x-4)\vec{j}+3xz^2\vec{k}$
5. If and $|\vec{r}| = r$, then $\nabla f(r) = \left[\frac{df(r)}{dr}\right]\hat{r} = f'(r)\hat{r}.$
6. Show that $\nabla^2(\phi\psi) = \phi\nabla^2\psi + 2\nabla\phi.\nabla\psi + \psi\nabla^2\phi.$
7. Show that $\nabla^2 \log r = \frac{1}{r^2}.$
8. Prove that $\vec{A} = (2x + yz)\vec{i} + (4y + xz)\vec{j} - (6z - xy)\vec{k}$, is solenoidal as well as irrotational. Also find the scalar potential of $\vec{A}$
9. Evaluate $\vec{f}\,d\vec{r}$ along the straight line joining (0, 0) and (1, 1)

(i)if $\vec{f} = x\vec{j} - y\vec{i}$

(ii)if $\vec{f} = x^2\vec{i} + y^2\vec{j}$.

10.Compute $\int_c \vec{F}.d\vec{r}$ along $y^2 = 4x$ from (0, 0) to (4, 4) if

(i) $\vec{F} = x\vec{i} - y\vec{j}$

(ii) $\vec{F} = 3x^2y\vec{i} + (x^3 - 2y^2)\vec{j}$

11. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$, $z = x^2 + y^2 - 3$ at the point (2,-1,2).

12. (i) Find the directional derivative of $\emptyset$ =xy+yz+zx in the direction of $\vec{i} + 2\vec{j} + 2\vec{k}$ at (1,1,3).

(ii) If $\vec{F} = xyz^2\vec{j} + x^2yz\vec{k}$ find div curl$\vec{F}$.

13. (i) If $\vec{F} = x^2y\vec{i} + y^2z\vec{j} + z^2x\vec{k}$, then find curl curl $\vec{F}$.

(ii) If $\vec{A}$ and $\vec{B}$ are vector point functions then prove that $\nabla.(\vec{A} \times \vec{B}) = (\nabla \times \vec{A}).\vec{B} - (\nabla \times \vec{B}).\vec{A}$.

14. Prove that $\nabla.(\vec{u} \times \vec{v}) = \vec{v}.(\nabla \times \vec{u}) - \vec{u}.(\nabla \times \vec{v})$.

15. Show that $\nabla \times (\nabla \times \vec{F}) = \nabla(\nabla.\vec{F}) - \nabla^2\vec{F}$.

16. If $\vec{A}$ is a vector point function, then prove that

(i)$\nabla.(\nabla \times \vec{A}) = 0$

(ii) $\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla.\vec{A}) - \nabla^2\vec{A}$.

17. (i) Find the angle between the normal to the surface xy-$z^2 = 0$ at the points (1,4,-2)and (-3,-3,3).

(ii) Find the directional derivative of xyz-xy^2z^3 at (1,2,-1) in the direction $\vec{\imath} - \vec{\imath} - \vec{\jmath} - 3\vec{k}$.

18. Evaluate $\iint_R$ xydxdy where R is the region enclosed by x=0, y=1, y=x.

19. If $\vec{F} = xyz\vec{\imath} + xyz^2\vec{\jmath} + x^2yz\vec{k}$, then find div curl $\vec{F}$.

20. Verify divergence theorem for $\vec{F} = x^2\vec{\imath} + y^2\vec{\jmath} + z^2\vec{k}$ taken over the cube bounded by the planes x=0, x=1, y=0, y=1, z=0, z=1.

21. If $\vec{F} = 3xy\vec{\imath} - y^2\vec{\jmath}$. compute $\int \vec{F}.\overline{dr}$ along the curve y=$2x^2$ from (0,0) to (1,2).

22. Find ∅ if ∇∅ is (y+sinz)$\vec{\imath} + x\vec{\jmath}$+xcosz$\vec{k}$

23. Find ∅ if ∇∅ is (6xy+z^3)$\vec{\imath} + (3x^2 - z)\vec{\jmath}$+$(3xz^2 - y)\vec{k}$.

Unit-4

VECTOR INTEGRATION

Two Marks

1. Define Integral theorems.
2. Define Vector integrals.

3.Using the Green's theorem, show that $\int_C$ (3x+4y)dx +(2x-3y)dy= -8π where *C* is the circle $x^2 + y^2 = 4.$

4. State Gauss' divergence theorem.

5.Show that, if *S* is a closed surface, then $\iint_s \vec{r}.\hat{n}dS = 3V$ where *V* is the volume enclosed by *S*.

6. Show that, if *S* is a surface of the sphere $x^2 + y^2 + z^2 = a^2$, then $\iint_s \vec{r}.\hat{n}dS = 4\pi a^3$.

7. Show that, if *S* is a surface of the rectangular parallelepiped formed by the planes $x = 0, \quad x = a; \quad y = 0, \quad y = b; \quad z = 0, \quad z = c$, then $\iint_s \vec{r}.\hat{n}dS = 3abc$.

8. Define Stroke's theorem.

9. State Green's theorem in the plane.

Five Marks

1. Using Green' theorem, show that $\int_c (3x^2 - 8y^2)dx + (4y - 6xy)dy = 20$, where C is the boundary of the rectangle formed by the lines $x=0, \quad x=1, y=0, \quad y=2$ in the xoy plane.

2. Evaluate by Green's theorem $\int_c (xy + x^2)dx + x^2 + y^2 dy$, Where C is the square formed by the lines $x=-1, \quad x=1, y=-1, \quad y=1$ in the xoy plane.

3. Evaluate by Green's theorem $\int_c e^{-x}(\sin y\, dx + \cos y\, dy)$, Where C is the rectangle with vertices $(0,0),(\pi,0),(\pi,\frac{\pi}{2}),(0,\frac{\pi}{2})$.

4. If S is a closed surface enclosing a volume v, then show that $\iint_s (ax\vec{i} + by\vec{j} + cz\vec{k}).\hat{n}ds = (a+b+c)v$.

5. If $\vec{A} = x^3\vec{i} + y^3\vec{j} + z^3\vec{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$, the show that $\iint_s \vec{A}.\overrightarrow{ds} = \frac{12}{5}\pi a^5$.

6. Evaluate $\iint_s$ x dydz + y dz dx +z dx dy over the surface of the sphere $x^2 + y^2 + z^2 = a^2$.

7. Evaluate $\iint_S x^3 dy\, dz + x^2\, y\, dz\, dx + x^2\, z$ dx dy, Where S is the surface of the cube bounded by the palnes x=0, y=0, z=0 ,x=a, y=a, z=a.

8. Using stroke's theorem ,

evaluate $\int_c$ (sin z dx- cos x dy+ sin y dz), where C is the boundary of the rectangle 0$\leq x \leq \pi$, 0$\leq y \leq 1$, z=3.

Ten Marks

1. Evaluate $\int_c$ xy dx-$x^2 dy$ by converting it into a double integral. It is given that C is the boundary of the region bounded by the line y=x and the parabola $x^2 = y$.

2. Verify Green's theorem for $\int_c$ (x-2y)dx +x dy, where C is the circle $x^2 + y^2 = 1$.

3. Verify Green's theorem for $\int_c$ (xy+y^2)dx+x^2 dy, where C is the closed curve of the region bounded by the line y=x and the parabola y=x^2.

4. Verify Green's theorem for $\int_C (3x^2 - 8y^2)dx+(4y-6xy)dy$,
   Where C is the boundary of the region enclosed by the parabola $x^2 = y$ and y^2=x.
5. Evaluate $\iint_S x^2 dy\, dz + y^2 dz\, dx + 2z(xy - x - y) dxdy$,
   where S is the surface of the cube
   $0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 1$.
6. Apply Gauss' divergence theorem to evaluate
   $\iint_S (x^3 - yz)\vec{\imath} - 2x^2 y\, dz\, dx + z\, dx\, dy$ over the surface of the cube bounded by the coordinate planes and the planes x=y=z=a.
7. verify divergence theorem for $\vec{F} = x^2\vec{\imath} + y^2\vec{\jmath} + z^2\vec{k}$ taken over the cube bounded by x=0,x=1;y=0,y=1;z=0,z=1.
8. verify divergence theorem for $\vec{F} = 4xz\vec{\imath} - y^2\vec{\jmath} + yz\vec{k}$ taken over the cube bounded by x=0,x=1;y=0,y=1;z=0,z=1.
9. verify stroke's theorem for $\vec{F} = (x^2 + y^2)\vec{\imath} + 2xy\vec{\jmath}$ taken over the rectangle bounded by x=0, x=a,y=0,y=b.
10. Verify stoke's theorem for the vector $\vec{F} = (x^2 - y^2)\vec{\imath} + 2xy\vec{\jmath}$ in the rectangular region in the xOy plane bouneded by the lines x=0, x=a, y=0, y=b.

11. Verify divergence theorem for $\vec{F} = 4xy\vec{\imath} - y^2\vec{\jmath} + yz\vec{k}$ taken over the cube x=0,y=0,y=1,z=0,z=1.

12. Verify stoke,s theorem for $\vec{A} = y\vec{\imath} + 2yz\vec{\jmath} + y^2\vec{k}$ taken over the upper half surfaces S of the sphere $x^2 + y^2 + z^2 = 1$,z=0.

13. Verify divergence theorem for $\vec{F} = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$ taken over the region bounded by x=y=z=0 and x=y=z=a.

14. Verify stoke,s theorem $\vec{F} = (2x - y)\vec{\imath} - yz\vec{\jmath} + y^2z\vec{k}$ where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.

15. Verify's Gauss's $\vec{F} = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$ taken over the region bounded by the plane x=0, x=a, y=0, y=a, z=0, z=a.

16. Verify stroke's theorem for $\vec{F} = x^2\vec{\imath} + xy\vec{\jmath}$ in the sequence region in xy planes bounded by the lines x=0, y=0, x=a, y=a.

17. Verify stoke,s theorem $\vec{F} = (x^2 + y^2)\vec{\imath} - 2xy\vec{\jmath}$ taken round bounded by the lines x=$\pm$a, y=0, y=b.

Unit-5

FINITE DIFFERENCES

Two Marks

1. Using Newton's method to find the smallest positive root of the equation $x^3-2x+0.5=0$.
2. Write the Newton's Backward Interpolation formula.
3. Write Newton's forward formula.
4. Prove that $1+\Delta = E$.

5. Define interpolation.
6. Define extrapolation.
7. Define Formula of interpolation.
8. Define Formula of extrapolation.
9. Define operator.
10. Define operator E.
11. Define Newton's forward formula.
12. Define Newton's backward formula.
13. Define $\Delta[f(x)]$. If h=1 Prove that $\Delta(\sin x) = 2\sin\frac{1}{2}\cos(x+\frac{1}{2})$.
14. Prove that $E\Delta = \Delta E$.
15. Write the forward difference table.
16. Write the relation between Δ *and* E.
17. Define Operator E^{-1}
18. Write the difference of a constant function.

Five Marks.

1. Find u_6, given that

u_0	u_1	u_2	u_3	u_4	u_5
25	25	22	18	15	15

2. Find the missing terms from the following data.

x	0	5	10	15	20	25
Y	7	11	14	-	24	32

3. Find the missing terms in the following tables.

X	1	2	3	4	5	6	7
Y	2	4	8	-	32	64	128

4. Find by suitable interpolation formula the value of f(2.5) from the following data.

X	2	3	4	5
f(x)	14.5	16.3	17.5	18

5. Using Newton's forward formula, find the value of y when x=21 from the following

X	20	23	26	29
Y	0.34	0.39	0.44	0.48

6. A function f(x) is given by the following table. Find f(0.2) by using Newton's forward interpolation formula.

X	0	1	2	3	4	5	6
f(x)	176	185	194	203	212	220	229

7. Using Newton's forward interpolation formula obtain y when x=2.5 from the following table.

X	0	1	2	3	4
Y	7	10	13	22	43

8. Given the table,

X	0	0.1	0.2	0.3	0.4
e^x	1	1.1052	1.2214	1.3499	1.4918

Find the value of $y=e^x$ when x=0.38 by using Newton's backward interpolation formula.

9. Using the following table, find f when x=0.25, using Newton's backward formula

X	0.1	0.2	0.3	0.4	0.5
f(x)	0.11	0.22	0.33	0.43	0.52

10. Using Newton's backward formula, find f(27.5) given

X	25	26	27	28
f(x)	16.195	15.919	15.630	15.326

11. Using Newton's backward formula, find f(1.9) given

X	1.00	1.25	1.50	1.75	2.00
f(x)	0.3679	0.2865	0.2231	0.1738	0.1353

12. Given the tabulated points (1,-3),(3,9),(4,30),(6,132), obtain the value of y when x=2 . Using Lagrange's interpolation formula.

Ten Marks

1. Find by Newton's method the real root of $x^3+3x-1=0$ correct to 2 decimal places.
2. Find the root of $x^4-x-10=0$ which is nearer to x=2 correct to three decimal places by Newton's method.
3. Find the positive root of the equation $x^3-2x^23x-4=0$ correct to 2 decimal places.
4. Using Newton's method, establish the formula, $x_n=1/2(x_{n-1}+N/x_{n-1})$ to find the square root of N. Hence find $\sqrt{29}$ correct to four places of decimals.
5. Using Newton's method, find the positive root of $x^3-2x-5=0$ correct ot 2 decimal places.

6. Obtain a first approximation to the real root of the equation $x^3=2x+5$ by Newton's method.
7. By Newton's method obtain a first approximation to the real root of $x^3+3x-7=0$.
8. Find by Newton's method the positive root of the equation $2x^3-3x-6=0$ which lies between 1 and 2.
9. Using Newton's method, obtain the formula $x_n=1/3(2x_{n-1}+N/x^2_{n-1})$ to find $N^{1/3}$. Find $29^{1/3}$ correct to three decimal places.
10. Diminish the roots of the following equation by 2: $x^4-5x^3+7x^2-4x+5=0$.
11. If $A=\begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}$. Find the eigen values of A^2.
12. Find the eigen values and eigen vectors of the $\begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}$.
13. Find the eigen values and eigen vectors of the $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$.
14. Find the eigen values and eigen vectors of the $\begin{pmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{pmatrix}$.
15. Find the eigen values and eigen vectors of the $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{pmatrix}$
16. Find the eigen values and eigen vectors of the $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$

17. Use Newton's formula to find y when x=142, given that

X	140	150	160	170	180
Y	3.685	4.854	6.302	8.076	10.225

18. Using Newton's formula, find the value of y when x=27, from the following data:

X	10	15	20	25	30
Y	35.4	32.2	29.1	26.0	23.1

19. Using Lagrange's formula, find $log_{10}301$ from the following table when x and log_{10}x values are given by

X	300	304	305	307
log_{10}x	2.4771	2.4829	2.4843	2.4871

20. Use Lagrange's formula to find y when x=2, given

X	0	3	5	6	8
Y	276	460	414	343	110

21. Use Lagrange's formula to find y when x=102, given

X	93.0	96.2	100	104.2	108.7
Y	11.8	12.80	14.7	17.07	19.91

22. The following table gives the values of x and y=$\sqrt{x}$.

X	1.00	1.05	1.10	1.15	1.20	1.25
Y	1.00	1.0247	1.04881	1.07238	1.09544	1.11803

23. Using Lagrange's formula find log_{10}401 from the following table where x, log_{10}x values are given.

X	400	404	405	407
log_{10}x	2.6021	2.6064	2.6075	2.6096

24. For a polynomial to the following data and hence find y(10) using Lagrange's interpolation formula

X	5	6	9	11
Y	12	13	14	16

25. By means of Lagrange's formula, show that $y_1 = y_3$-0.3($y_5 - y_{-3}$)+0.2($y_{-3} - y_{-5}$).

26. Given u_0 =-4, u_1 =-2,$u_{4=}$220,u_5=546,u_6 =1148, find u_2 andu_3

27. Using Newton's formula find the value of y when x=27 from the following data.

X	10	12	20	25	30
Y	35.4	32.2	29.1	26.0	23.1

28. Use Lagrange's formula to find y when x=2 given

X	0	3	5	6	8
y	276	460	414	343	110

29. Using Newton's formula find y when x=27, from the following data:

x:	10	15	20	25	30
y:	35.4	32.2	29.1	26.0	23.1

30. The following data give the percentage of criminals for different age groups:

Age(less than x)	:	25	30	40	5
Percentage of criminals	:	52.0	67.3	84.1	94.4

31. Using Lagrange's formula estimate the percentage of criminals under the age of 35.

From the following table, find the value of tan 45^o15'

X:	45	46	47	48	49	50
Y:	1.00000	1.03553	1.07237	1.11061	1.15037	1.19175

32. From the following table, find the vale θ at x=43 and x-84.

X:	40	50	60	70	80	90
Y:	184	204	226	250	276	304

33. Using Newton's backward formula find f(27.5) given

X:	25	26	27	28
Y:	16.195	15.919	15.630	15.326

34. Use Lagrange's formula to find y when x=2

X:	0	3	5	6	8
Y:	276	460	414	343	110

35. Using Newton's forward interpolation formula, find the value of y when x=21 from the following tabulated values of the function:

X:	20	23	26	29
Y:	0.3420	0.3907	0.4384	0.4848

36. Given the values

X :	14	17	31	35
f(x):	68.7	64.0	44.0	39.1

37. Find the values of f(x) corresponding to x=27 using Lagrange's formula.

From the following table of half yearly premium for policies maturing at different ages, estimate the premium for policies maturing at age 46 and 63.

X: 20 23 26 29

Y: 0.3420 0.3907 0.4384 0.4848

38. Find the value of y when x=8 from the following data, using newton's forward method.

X	0	5	10	15	20	25
Y	7	11	14	18	24	32

39. Use Lagrange's formula to find y when x=102, given

X	93.0	96.2	100	104.2	108.7
Y	11.8	12.80	14.7	17.07	19.91

40. Use Newton's formula to find y when x=142, given that

X	140	150	160	170	180
Y	3.685	4.854	6.302	8.076	10.225

41. Use Lagrange's formula to find y when x=2, given

X	0	3	5	6	8
Y	276	460	414	343	110

42. Using Newton's backward formula find f(1.9) given

X	1	1.25	1.5	1.75	2.00
Y:	0.3679	0.2865	0.2231	0.1738	0.1353