St. Joseph's College of Arts and Science, Cuddalore.

Question Bank

PG Research Department of Mathematics

Class: I B.Sc (Physics and Chemistry)

Subject Name: Allied Mathematics II

Subject Code: AMT202S

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Semester : II

Unit -1

3- marks

1. If x= cosθ+isinθ, what is $(x - 1/x)^n$?

2. If x= cosθ+isinθ, write the expansion of x^4-1/x^4.

3. Evaluate the following:

   (i) $\lim_{n\to 0} \frac{x-\sin x}{x^3}$

   (ii) $\lim_{n\to 0} \frac{\tan x-\sin x}{x^3}$.

   (iii) $\lim_{n\to 0} \frac{\sinh x-\sin x}{x^3}$.

4. If $\frac{\sin\theta}{x} = \frac{2165}{2166}$ show that θ is equal to 3° 1' nearly.

5. Find the logarithm of i.

6- marks:

1. Express cos 6θ as a polynomial in cos θ.
2. Express cos 6θ as a polynomial in sin θ
3. show that cos 8θ= 128 $\cos^8\theta$+160$\cos^4$-32 $\cos^2\theta$+1
4. Express sin 6θ/sin θ as a polynomial in cosθ.
5. Express sin 6θ/sin θ as a polynomial in sinθ
6. S

   how that sin 6θ/sin θ=32 $\cos^5\theta$-32 $\cos^3\theta$+6 cosθ.
7. Express cos 5θ\cos θ as a polynomial in

   (i) Cos θ

   (ii) Sin θ.
8. W

   rite the expansion of tan nθ.
9. W

   rite the expansion of tan($\theta_1+\theta_2+\ldots+\theta_n$).
10. If ∝, β, γ are the roots of the equation $x^3+px^2+qx+p=0$ prove that

    $\tan^{-1}\alpha+\tan^{-1}\beta+\tan^{-1}\gamma=n\pi$.
11. Find the approximate values of θ in radian if

    (i) $\frac{\sin\theta}{\theta}$ =863/864,

    (ii) $\frac{\tan\theta}{\theta}$ =2524/2523.
12. Show the following results:
13. $\lim_{n\to 0}\frac{\tan 2x-}{4x}$

    =3/4.

14. $\lim_{n\to\pi/2} \frac{\cos x}{c\ldots}$

=1/3.

15.

Finding the real and imaginary parts

(i)sin(θ+i∅), sin(θ-i∅)

(ii)tan(θ+i∅), tan(θ-i∅).

16.

Expand sin 7θ as a polynomial in sinθ. Hence show that

sin π/7.sin 2π/7.sin 3π/7.sin 4π/7.sin5π/7.sin6π/7=-7/64.

17.

If $\tan^{-1}$ x+$\tan^{-1}$y+$\tan^{-1}$z=π\2,show that xy+yz+zx=1.

18.

Show that series for tan x as far as the term x^5.

19.

Expansions of sin θ, cos θ, tan θ in θ.

20.

Relations between circular and hyperbolic functions.

10 marks

1. Show that 1. 2^5 $\cos^6$θ=cos 6θ+6cos 4θ+15 cos 2θ+10,

2. Show that.$2^6\cos^7$θ=cos 7θ+7 cos 5θ+21 cos 3θ+35 cosθ.

2. Prove the results in the following sums:

$2^7\cos^8$θ=cos 8θ+ 8 cos 6θ+28 cos 4θ+56 cos 2θ+35.

3. Prove the results in the following sums:

$2^8\cos^5$θ $\sin^4$θ=cos 9θ+cos 7θ-4 cos 5θ-4 cos 3θ+ 6 cos θ.

4. Express sin 5θ/sin θ as a polynomial in (i) Cosθ and (ii) Sin θ.

5. Express sin 7θ/sin θ as a polynomial in (i) Cosθ,(ii) Sinθ.

6. Prove the following:

$\cos 8\theta = 1-32\sin^2\theta+160\sin^4\theta-256\sin^6\theta+128\sin^8\theta$.

7. Express sin 9θ\cosθ in terms of sin θ.

8. If tan x\2=tan hx\2, show that cos x cos hx =1.

9. If tan (θ+i∅)= cos ∝+isin ∝, then show that

(i) tan 2∅=sin ∝, (ii) cos h2∅=sec∝.

10. Prove that hyperbolic functions.

11. If sin(A+iB)=x+iy, then

1. Show that x=sin A cosh B,

2.show that $x^2\backslash\sin^2 A-y^2/\cos^2 A=1$,

3. Show that $x^2\backslash/+/\backslash\sinh^2 B=1$.

12. If log (sin θ+i∅)=A+iB=A+iB, prove that $2e^{2A}$=cos 2∅-cos2θ.

13. If sin(x+iy)= tan (θ+i∅), show that

1. Sin 2θ cot x=sinh 2∅ coth y,

2. Sin 2θ/sin 2∅ = tan x/tanh y.

14.If sin(x+iy) =r (cos θ+isinθ), then show that,

1. r^2=1/2[cosh 2y-cos 2x],

2. tanθ=cot x tanh y.

15. If log [sin(θ+i∅)]=A+iB, prove that $2e^{2a}$=cosh 2∅-cos 2θ.

Unit -2

3 Marks:

1. Eliminate the arbitrary constants a, b from z=ax+by to get an equation in x, y, z.
2. Eliminate the arbitrary function f from z=f(x^2+Y^2).
3. Eliminate the arbitrary function f from f(x^2+y^2,z-xy)=0
4. Solve $\sqrt{p}$+$\sqrt{q}$=x.
5. Solve pq=y.

6 Marks:

1. From a partial differential equation by eliminating the constants a and b from the equation z=(x+a)(y+b)
2. Eliminate the arbitrary function f from z=e^y f(x+y).
3. Eliminate the arbitrary function f from f(xy+z^2,x+y+z)=0.
4. Eliminate the arbitrary function f from f(x^2+y^2,z-xy)=0.
5. Eliminate the arbitrary function f from f(x^2+y^2+z^2,z^2-2xy)=0
6. Eliminate the arbitrary function f_1 and f_2 from z=f_1(x+y)+f_2(x-y).
7. Eliminate f and ∅fromz=(x+ay)+∅(x-ay).
8. Eliminate f and g from z=f(x+ay)+g(x+by).
9. Solve p^2+q^2=npq.
10. Solve pq+p+q=0.
11. Solve xypq=z^2.
12. Solve xp+p^2=q.
13. Solve z^2(p^2+q^2+1)=1.
14. Solve z^4q^2-z^2p=1.

15. Solve pz=1+q^2

16. Solve q-p+x-y=0.

17. Solve the equations:

1.pq=z,

2.p(1+q)=qz,

3. Zpq=p+q,

4. Z=p^2-q^2

18. Solve the following:

1.pq=x,

2. P=2qx,

3. $p^2(1+x^2)$=q.

19. Solve x+y $\partial z/\partial x$=0.

20. Solve $\partial^2/\partial x\partial y=x^2+y^2$.

10- Marks:

1. Eliminate the arbitrary function f from

(i) Z=x+y+f(xy),

(ii) z=f(y/x), 3.z=xf(y/x),

(iii) z=f(xy/z),

(iii) f($x^2+y^2+z^2$, X+y+z)=0.

2. Solve $(x/p)^n+(y/q)^n=z^2$.

3. Solve p=$(1+q^2)y^2$.

4. Solve the equation $z^2(p^2+q^2)=x^2+y^2$.

5. Solve the equations xp+zq=y.

6. Solve the equation (x^2-y^2-z^2)p+2xyq=2zx.

7. Solve (y-z)p+(z-x)q=x-y.

8. Solve x(y-z)p+y(z-X)q=z(x-Y).

9. Solve x(z^2-y^2)p+y(x^2-z^2)q=z(y^2-x^2).

10. Solve the equation x(y^2-z^2)p+y(x^2-z^2)q=z(y^2-x^2).

11. Solve (mz-ny)p+(nx-lz)q=ly-mx.

12. Solve (x^2-yz)p+(y^2-zx)q=z^2-xy.

13. Find the integral surface of x(y^2+z)p-y(x^2+z)q=(x^2-y^2)z

14. Solve the following equations

(i) Ap+bq+cz=0,

(ii) Xp+yq=z,

(iii) P-q=sin(x+y).

15. Solve the following equations:

(i) x^2p+y^2q=(x+y)z,

(ii) Z(xp-yp)=y^2-x^2.

16. Solve the equations:

(i)xzp+yxq=xy,

(ii)(y^2+z^2)p-xyq=-xz.

17. Find the general solutions of

(i) (y+z)p+(z+x)q=x+y,

(ii)(y+z-x)p+(z+x-y)q=x+Y-z,

(i) (y+zx)+(x+yz)=x^2-y^2.

18. Find the integral surface of the partial differential equation xp+yp=z, passing through the parabola y^2=4x, z=1.

19. Show that the integral surface satisfying 4yzp+q+2y=0 and passing through the curve x+z=2, y^2+z^2=1 is y^2+z^2+x+z=3.

20. Solve the following equations (y^3x-2x^4)p+(2y^4-x^3y)q=9z(x^3-y^3).

Unit-3

3- Marks

1. Define Scalar and vector point functions.
2. Define Vector Point functions.
3. State angle between two surfaces.
4. State unit vector normal to the surface.
5. Find the directional derivative of the function $x^2+y^2+z^2$ at (3,6,9) in the direction whose d.c.'s are 1/3, 2/3, 2/3.
6. Find the unit vector normal to the surface $x^2+y^2+2z^2=4$.
7. Find $\nabla\phi$ if
   (i) $\phi=x^2y^3z^2$
   (ii) $\phi=xyz-x^2$
8. Find $\nabla\phi$ in the following cases at the points specified:
   (i) ϕ(x, y, z)=2xy-y^2 at (1, 3, 2)
   (ii) ϕ(x, y, z)=x=xy^2+yz^2 at (1, 0, 0)
   (iii) ϕ(x, y, z)=y^2(x-z) at (1, 1, 2)
9. Define solenoidal Vector.

10. Define Irrotational Vector.

11. Show that the Vector $\vec{A}=x^2z^2\vec{i}+xyz^2\vec{j}-xz^3\vec{k}$ solenoid.

12. Determine the constant a so that the vector $\vec{F} = (x+2y)\vec{i} + (y-2z)\vec{j} + (x+az)\vec{k}$.

13. Show that $\vec{F} = yz\vec{i} + zx\vec{j} + xy\vec{k}$ is irrotational.

14. If $\vec{A} = axy\vec{i} + (x^2 + 2yz)\vec{j} + y^2\vec{k}$ is irrotational, find the value of a.

15. If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, show that $\nabla.\vec{r} = 3$.

16. Show that $x^2\vec{i} + 2xy\vec{j} - 4xz\vec{k}$, is solenoidal.

17. Find the values of m if the following vectors are solenoidal:

(i) $\vec{F} = (x+2y)\vec{i} + (my+4z)\vec{j} + (5z+6x)\vec{k}$,

(ii) $\vec{F} = (2x+y)\vec{i} + (4x-11y+3z)\vec{j} + (3x+mz)\vec{k}$,

(iii) $\vec{F} = (x+3y)\vec{i} + (y-2z)\vec{j} + (x+mz)\vec{k}$.

18. Show that $\vec{A} = (4xy - z^3)\vec{i} + 2x^2\vec{j} - 3xz^2\vec{k}$ is irrotatioanl.

19. If $\vec{r} = x\vec{i} + y\ \vec{j} + z\vec{k}$ and $\vec{r} = r\hat{r}$, show that (i)

6- Marks

1. Prove that the directional derivative of $\phi = x^3 y^2 z$ at (1,2,3) is a maximum along the direction $\vec{9i} + \vec{3j} + \vec{k}$. Find this maximum directional derivative.
2. Find the directional derivative of the following scalar point functions at the given points in the given directions.

Si.	Function ϕ	Point	Direction

No.			
(i)	xy+yz+zx	(1, 1, 3)	$\vec{i}+2\vec{j}+2\vec{k}$
(ii)	Xyz	(2, 1, 1)	$\vec{j}+\vec{k}$
(iii)	$x^2yz+4xz^2$	(1, -2, -1)	$2\vec{i}-\vec{j}-2\vec{k}$
(iv)	2xy+5yz+zx	(1, 2, 3)	$3\vec{i}-5\vec{j}+4\vec{k}$

3. Find the angle between the surfaces $x^2+y^2+2z^2=4$, $z=x^2+y^2-3$ at the point (2, -1, 2).
4. Find the equation of the tangent plane to the ellipsoid $x^2+2y^2+3z^2=6$ at (1, 1, 1).
5. Find the directional derivatives of ϕ at the given point in the directional of the given vector.

Si. No	Functions ϕ	Point	Vector
(i)	$2x^2+3y^2+z^2$	(2, 1, 3)	$\vec{i}-2\vec{k}$
(ii)	xy^2	(1, 1, 0)	$\vec{i}+\vec{j}+\vec{k}$
(iii)	Yz^2	(0, 1, 1)	$\vec{i}+\vec{j}+\vec{k}$
(iv)	$3xy^2-x^2yz$	(1, 2, 3)	$\vec{i}-2\vec{j}+2\vec{k}$
(v)	$xyz-xy^2z^3$	(1, 2, -1)	$\vec{i}-\vec{j}-3\vec{k}$
(vi)	$X^3+y^3+z^3$	(1, -1, 2)	$\vec{i}+2\vec{j}+\vec{k}$
(vii)	$4xz^2+2xyz$	(1, 2, 3)	$2\vec{i}+\vec{j}-\vec{k}$
(viii)	x^2y^2z	(2, 1, 4)	$\vec{i}+2\vec{j}+2\vec{k}$
(ix)	$x^2yz+4xz^2+xyz$	(1, 2, 3)	$2\vec{i}+\vec{j}-\vec{k}$

6. Find the unit vectors $\hat{n}$ normal to the following surfaces at the specified points.
   (i) $x^2+2y^2+z^2=7$ at (1, -1, 2)
   (ii) $x^2+y^2-z^2=1$ at (1, 1, 1)
   (iii) $x^2+3y^2+2z^2=6$ at (2, 0, 1)
   (iv) $x^2+y^2-z=1$ at(1, 1, 1)
   (v) $x^3-xyz+y^3=1$ at (1, 1, 1)
7. Find the maximum value of directional derivative of the function $\phi=2x+3y^2+5z^2$ at the point (1, 1, -4).

8. Find the equation of the tangent plane to the surface $x^2+2y^2+3z^2=6$ at (1, -1, 1)
9. Find the angle between the normal to the surface $xy-z^2=0$ at the points (1, 4, 2) and (-3, -3, 3).
10. Prove that curl $((\vec{r}\times\vec{a}) = -2\vec{a}$, where $\vec{a}$ is a constant vector.

11. Find Curl $\vec{F}$ in the following cases:

(i) $\vec{F} = x^2 y\vec{i} + y^2 z\vec{j} + z^2 x\vec{k}$

(ii) $\vec{F} = xyz\vec{i} + xyz^2\vec{j} + x^2 yz\vec{k}$

12. Find the value of the constants a, b, c so that
$\vec{F} = (x+2y+az)\vec{i} + (bx-3y-z)\ \vec{j} + (4x+cy+2z)\vec{k}$

13. If $\vec{F} = xyz\vec{i} + xyz^2\vec{j} + x^2 yz\vec{k}$, then find div curl $\vec{F}$.

14. If $\vec{F} = x^2 y\vec{i} + y^2 z\vec{j} + z^2 x\vec{k}$, then find div curl $\vec{F}$.

15. Find $\nabla\times\vec{F}$ if $\vec{F} = xz^3\vec{i} - 2x^2 yz\vec{j} + 2yz^4\vec{k}$.

16. Find the curl of the following functions at the specified points.

(i) $x^2 z\vec{i} - 2y^3 z^2\vec{j} + xy^2 z\vec{k}$ at (1, -1, 1)

(ii) $xyz\vec{i} + 3x^2 yz\vec{j} + (xz^2 - y^2 z)\vec{k}$ at (1, 2, -1)

17. Find a if $\vec{A} = (4xy - z^3)\vec{i} + ax^2\vec{j} - 3xz^2\vec{k}$ is irrotational.

10 –Marks

1. Find ϕ(x, y, z) given that ϕ(1, 1,1)=3 and
   $\nabla\phi = (y + y^2 + z^2)\vec{i} + (x + z + 2xy)\vec{j} + (y + 2zx)\vec{k}.$
2. Find the angle between the surfaces $x^2+yz=2$ and $x+2y-z=2$ at (1, 1, 1).
3. Find the angle between the surfaces $x^2+y^2=4-5z$ and $x^2+y^2+3z^2=104$ at (5, 2, -5).

4. Find ϕ if $\nabla\phi$ is
   (i) $(6xy+z^3)\vec{i} + (3x^2-z)\vec{j} + (3xz^2-y)\vec{k}$
   (ii) $(2yz+1)\vec{i} + x^2z\vec{j} + x^2y\vec{k}$
   (iii) $(y+\sin z)\vec{i} + x\vec{j} + x\cos z\vec{k}$
   (iv) $(y^2\cos x+z^3)\vec{i} + (2y\sin x-4)\vec{j} + 3xz^2\vec{k}$
5. If $\vec{r} = \vec{xi} + \vec{yj} + \vec{zk}$ and $|\vec{r}| = r$, then $\nabla f(r) = \left[\frac{df(r)}{dr}\right]\hat{r} = f'(r)\hat{r}.$

Unit 4

3 Marks

1. State Green's theorem
2. State stroke's theorem.
3. State gauss theorem.
4. Evaluate the Green's theorem $\int_c (xy+x^2)dx+(x^2+y^2)dy$, where C is the square formed by the lines x=-1, x=1, y=-1,y=1 in the xOy plane.
5. Using the Green's theorem, show that $\int_c (3x^2-8y^2)dx+(4y-6xy)dy=20$.

6- Marks:

1. Evaluate by Green's theorem $\int_c e^{-x}(\sin y\, dx+\cos y\, dy)$, where C is the rectangle with vertices (0,0), (π,0), (π,π\2),(0,π\2).

2. Evaluate $\int_c$xy dx-x^2 dy by conveerting it into a double integral.It is given that C is the boundry of the region bounded by the line y=x and the parabola x^2=y.
3. Verify Green's theorem for $\int_c$ (x-2y)dx+xdy, where C is the circle $x^2+y^2=1$.
4. Verify Green's theorem for $\int_c x^2(1+y)dx+(y^3+x^3)dy$, where C is the square formed by the lines y=$\pm$1, x=$\pm$1.
5. If Ā=$x^3+y^3\bar{j}+z^3\vec{k}$ and s is the surface of the sphere$x^2+y^2+z^2=a^2$,then show that$\int\int_s$ Ā.$\overrightarrow{ds}$=12/5 πa^5
6. If $\vec{A}=\overline{x\vec{\imath}}+\overline{y\vec{\jmath}}$-2z$\vec{k}$ and s is the curved surface of the upper hemisphere $x^2+y^2+z^2=a^2$, then show that ∬s Ā.$\hat{n}$ ds=0.
7. If $\vec{A}$=6z$\vec{\imath}$+ (2x+y)$\vec{\jmath}$-x$\vec{k}$ and s is the surface of the cylindrical solid bounded by y=0,y=8;x=0,z=0;x^2+z^2=9,then show that$\int\int_s$ Ā.$\hat{n}$ ds=18π.
8. Evaluate $\int\int_s$ x dy dz +y dz dx+ z dx dy over the surface of the sphere $x^2+y^2+z^2=a^2$.
9. Evaluate $\int\int_s x^3$ dy dz+x^2 y dx+x^2 dx dy, where S is the surface of the cube bounded by the planes x=0, y=0, z=0, x=a, y=a,z=a.

10. Apply Gauss divergence theorem to evaluate $\int\int_s(x^3\text{-yz})$dy dz-2$x^2$ y dz dx+ z dx dy over the surface of the cube bounded by the cooridate planes and the planes x=y=z=a.

11. Show that, for any closed surface S, $\int\int_s(\emptyset\nabla\Psi-\Psi\nabla\emptyset).\ d\bar{s}=\int\int\int_v \nabla.\ (\emptyset\nabla\Psi)dV$.

12. Show that the volume bounded by a closed surface S is 1\3 $\int\int_s\bar{r}.\hat{n}$ dS.

13. If S is the surfave of the sphere $x^2+y^2+z^2$=9, then show that $\int\int_s\bar{r}.\hat{n}$ dS=108π.

14. If S is the surface of the rectangular parallelopiped formed by the planes x=-1, x=1, y=-2, y=2, z=-3, z=3, show that $\int\int_s \bar{r}.\hat{n}$ dS=144.

15. Evaluate $\int\int_s\bar{A}.\hat{n}$ Ds, if $\bar{A}$ =2xy$\bar{\imath}$+y$z^2\bar{\jmath}$+xz$\bar{k}$ and S is the surface of the parallelopiped formed by the planes x=0, x=2, y=0, y=1, z=0, z=3.

16. Apply divergence theorem to evaluate $\int\int_s$(x+z) dy dz+(y+z)dz dx+(x+y) dx dy, where S is the surface of the sphere $x^2+y^2+z^2=a^2$.

17. Verify Gauss' divergence theorem for $\bar{F}=x^2\bar{\imath}+2y^2\bar{\jmath}+3z^2\bar{k}$ taken over the cube bounded by the planes x=0, x=1, y=0, y=1, z=0, z=1.

18. Using Stoke's theorem, evaluate $\int_c$(sin zdx-cos x dy+sin y dz), where C is the boundary of the rectangle 0≤x≤π, 0≤y≤1, z=3.

19. Verify stoke's theorem for $\bar{F}=(x^2+y^2)\bar{\imath}+2xy\bar{\jmath}$ taken over the rectangle bounded by x=0, x=a, y=0, y=b.

20. Verify Stoke's theorem for $\bar{A}=y\bar{\imath}+2yz\bar{\jmath}+y^2\bar{k}$ taken over the upper half surface S of the sphere $x^2+y^2+z^2=1$, $z\geq0$ and the vounding cirlce C, $x^2+y^2=1$, z=0.

10 Marks:

1. Prove that the area enclosed by a simple closed curve C is $1/2\int_c x\,dy-y\,dx$, 2. Evaluate $\int_c x\,dy-ydx$, where C is the circle $x^2+y^2=4$.
2. Verify Green's theorem for $\int_c(xy+y^2)dx+x^2dy$, where C is the closed curve of the region bounded by the line y=x and the parabola $y=x^2$.
3. Verify Green's theorem for $\int_c(3x^2-8y^2)dx+(4y-6xy)dy$, where C is the boundry of the region enclosed be the parabolas $x^2=y$ and $y^2=x$.
4. Verify Green's theorem for $\int_c(3x^2-8y^2)dx+(4y-6xy)\,dy$, where C is the boundry of the region R enclosed by the straight lines y=0,x+y=1, x=0.
5. Verify Green's theorem for $\int_c x^2(1+y)dx+(y^3+x^3)dy$, where C is the square formed by $x=\pm a, y=\pm a$
6. Evaluate $\int\int_s x^2dy\,dz+y^2dz\,dx+2z(xy-x-y)\,dx\,dy$, where S is the surface of the cube $0\leq x\leq1$, $0\leq y\leq1$, $0\leq z\leq1$.

7. Verify divergence theorem for $\bar{F}=x^2\bar{\imath}+y^2\bar{\jmath}+z^2\bar{k}$ taken over the cube bounded by the planes x=0, x=1, y=0,y=1, z=0, z=1 now we have to verify that $\int\int_s \bar{F}.\hat{n}$ ds=$\int\int\int_v \nabla.\bar{f}$dV.

8. Verify divergence theorem for $\bar{F}$=4xz$\bar{\imath}$-$y^2\bar{\jmath}$+yz$\bar{k}$ taken over the cube bounded by x=0, x=1, y=0, y=1, z=0, z=1, now we have to verify that $\int\int_s \bar{F}.\hat{n}$ dS=$\int\int\int_v \nabla.\bar{F}$ dV.

9. Verify divergence theorem for $\bar{A}$=4x$\bar{\imath}$-2$y^2\bar{\jmath}$+$z^2\bar{k}$ and the surface S of the cylinder bounded by z=0, z=3, $x^2+y^2=4$.

10. Verify divergence theorem for $\bar{A}$=a(x+y)$\bar{\imath}$+a(y-x)$\bar{\jmath}$+$z^2\bar{k}$. taken over the region bounded by the upper hemisphere $x^2+y^2+z^2=a^2$ and the plane z=0.

11. Verify Gauss' divergence theorem for $\bar{f}$=(x^3-yz)$\bar{\imath}$-2x^2y$\bar{\jmath}$+2$\bar{k}$ over the cube bounded by x=0, y=0, z=0, x=a, y=a, z=a, now we have to verify that $\int\int_s \bar{f}.\hat{n}$ dS=$\int\int\int_v \nabla.\bar{f}$ dV.

12. Apply divergence theorem to evaluate $\int\int_s$(x+z)dy dz+(y+z)dz dx+(x+y)dx dy, where S is the surface of the sphere $x^2+y^2+z^2=a^2$.

13. Verify Gauss' divergencce theorem for $\bar{F}=x^2\bar{\imath}+2y^2\bar{\jmath}+3z^2\bar{k}$ taken over the cube bounded by the planes x=0, x=1, y=0, y=1, z=0, z=1.

14. Verify Gauss' divergence theorem for $\bar{F}=x^3\bar{\imath}+y^3\bar{\jmath}+z\bar{k}$ taken over the cube bounded by x=0, x=a, y=0, y=a, z=0, z=a.

15. Verify Gauss' divergence theorem for $\bar{A}=(x+y)\bar{\imath}+x\bar{\jmath}+z\bar{k}$ and the bounded by the planes x=0, x=1, y=0, y=1, z=0, z=1.

16. Verify divergence theorem for $\bar{A}=2xz\bar{\imath}+yz\bar{\jmath}+z^2\bar{k}$ and the region bounded by the upper half of the sphere $x^2+y^2+z^2=a^2$ and the plane z=0.

17. Verify Stoke's theorem for the vector $\bar{F}=(x^2-y^2)\bar{\imath}+2xy\bar{\jmath}$ in the rectangular region in the xOy plane bounded by the lines x=0, x=a, y=0, y=b.

18. Verify Stoke's theorem for $\bar{F}=(x^2+y^2)\bar{\imath}+2xy\bar{\jmath}$ taken over the rectangle bounded by x=0, x=a, y=0, y=b.

19. Verify Stoke's theorem for $\bar{A}=x^2\bar{\imath}+xy\bar{\jmath}$ taken over the square S in the xy plane whose vertices are o (0,0,0), A(a,0,0), B(a,a,0), c(0,a,0) and oveer its boundary.

20. Verify stoke's theorem for the function $\bar{A}=x^2z\bar{\imath}+xy\bar{\jmath}$ integrated along the rectangle in the plane z=0, where the sides are along the lines x=0, y=0, x=a, y=b.

Unit- 5

3 marks:

1. Define relation between Δ and E.

2. Define operator E^{-1}.

3. Write newton's forward formula.

4. Write newton's backward formula.

5. Write lagrange's formula.

Derivations of Newton's forward formula.

6 marks:

1. Find the missing y_x values in the table

y_x	0	_	_	_	_	_
Δy_x	0	1	2	4	7	11

2. Find u_6, given that

u_0	u_1	u_2	u_3	u_4	u_5
25	25	22	18	15	15

3. Find the missing term from the following data.

X	0	5	10	15	20	25
Y	7	11	14	_	24	32

4. Find the missing terms in the following table:

X	1	2	3	4	5	6	7
Y	2	4	8	_	32	64	128

5. Define Δ[f(x)]. If h=1, prove that Δ(sin x)=2 sin 1/2 cos [x+1/2].

6. Find the suitable interpolation formula the values of f(2.5) from the following data.

X	2	3	4	5
Y	14.5	16.3	17.5	18

7. Using Newton's forward interpolation formula obtain y when x=2.5 from the following table.

X	0	1	2	3	4
Y	7	10	13	22	43

8. Give the table,

X	0	0.1	0.2	0.3	0.4
e^x	*1*	1.1052	1.2214	1.3499	1.4918

9. Given the tabulated points (1,-3), (3,9), (4,30), (6,132), obtain the values of y when x=2 using Largrange's interpolation formula.

10. Using Newton's forward interpolation formula, find the population for the year 1946given

X (year)	1941	1951	1961	1971	1981
Y (popolation in thousands)	46	66	81	93	101

11. Given the following values for x and y

X	0	1	2	3	4	5
Y	3	12	81	200	100	8

Find $\Delta^5 y_0$.

12. Given

X	40	50	60	70	80	90
Y	184	204	226	250	276	304

Find y, when x=43, using Newton's forward difference formula.

13. Using Newton's interpolation formula, find U_8 given

U_7=351,U_8=739, U_{11}=1343, U_{13}=2211

14. Derivation for finding polynomial function.

10 marks:

1. Apply Newton's backward difference formula to find a polynomial of degree 3. Using the table given below.

X	3	4	5	6
Y	6	24	60	120

2. Use Newton's formula to find y when x=142, given that

X	140	150	160	170	180
Y	3.685	4.854	6.302	8.076	10.225

3. It is given that

X	40	50	60	70	80
Y	3.7	4.9	6.3	8	10.2

Find the value of y corresponding to x=45, suing Newton's formula.

4. Find the value of y when x=8 from the following data:

X	O	5	10	15	20	25
y	7	11	14	18	24	32

5. Using Newton's formula, find the value of y when x=27, from the following data:

X	10	15	20	25	30
Y	35.4	32.2	29.1	26.0	23.1

6. Using Lagrange's formula, find log_{10}301 from the following table when x and log_{10}x values are given by

X	300	304	305	307

log_{10}x	2.4771	2.4829	2.4843	2.4871

7. Use Lagrange's formula to find y when x=2, given

X	0	3	5	6	8
Y	276	460	414	343	110

8. Use Lagrange's formula to find y when x=102, given

X	93.0	96.2	100	104.2	108.7
Y	11.38	12.80	14.7	17.07	19.91

9. Using Newton's formulae, calculate the values of $e^{1.85}$ and $e^{2.05}$ given:

X	1.7	1.8	1.9	2.0	2.1
e^x	5.474	6.050	6.686	7.389	8.166

10. Using Newton's forward formula, find the vales of y when x=21 from the following,

X	20	30	40	50
Y	0.34	0.39	0.44	0.48

11. A function f(x) is given by the following table. Find f(0.2) by using Newton's forward interpolation formula,

X	0	1	2	3	4	5	6
F(x)	176	185	194	203	212	220	229

12. Using Lagrange's formula, find log_{10}401 from the following table where x, log_{10}x values are given:

X	400	404	405	407
log_{10}x	2.6022	2.6064	2.6075	2.6096

13. For a polynomial to the following date and hence find y(10) using Lagrange's interpolation formula

X	5	6	9	11
Y	12	13	14	16

14. Using the following table, find y when x=0.25, using Newton's backward formula

X	0.1	0.2	0.3	0.4	0.5
F(x)	0.11	0.22	0.33	0.43	0.52

15. Using Newton's backward formula, find (27.5) given

X	25	26	27	28
F(x)	16.195	15.919	15.630	15.326

16. Using Newton's backward formula, find f(1.9) given

X	1.00	1.25	1.50	1.75	2.00
F(x)	0.3679	0.2865	0.2231	0.1738	0.1353

17. The following table gives the values of x and y =$\sqrt{x}$

X	1.00	1.05	1.10	1.15	1.20	1.25
Y	1.00	1.0247	1.04881	1.07238	1.09544	1.11803