Department of Mathematics

QUESTION BANK

Class: II M.Sc Mathematics

Sub Name: Graph theory

Sub Code: EPMT1020

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-QUESTIONS

2MARKS

UNIT -I

1. State the shortest path problem.
2. Define trial
3. Define walk with an example
4. Define weighted graph.

UNIT –II

1. Prove that every connected graph contains a spanning tree.
2. Define tree .
3. Define cut edge.

4. Define a bond.

UNIT-III

1. State Konigsberg bridge problem.
2. Define Eulerian graph.
3. Define Hamiltonian graph.
4. Draw the Herschel graph and State whether it is Hamiltonian or not?

UNIT-IV

1. Let G be a bipartite , prove that $\chi' = \Delta$
2. Find $r(3,3)$
3. Define edge chromatic number.
4. Draw the Petersen graph and find its edge chromatic number.

UNIT-V

1. Given an example of a 4-critical graph.
2. Define connectivity.
3. When does a graph G is said to be k-colorable?
4. State Halos' conjecture.
5. Define k-colourable graph.
6. Define k-critical graph.
7. Prove that Every critical graph is a block.
8. Define exterior of a Jordan curve.
9. Define S-Component.
10. Define stereographic projection.
11. Define planar graphs and embeddable.

5 MARKS

UNIT-I

1. State and prove sperner's lemma
2. State if G is simple and $\varepsilon > [\frac{\nu-1}{2}]$ then prove that G is connected
3. State if G is disconnected then G^c is connected
4. State that any two longest paths in a connected graph have a vertex in common.

UNIT –II

1. Prove that if G is a tree,then $\epsilon = \nu + 1$
2. Let G is a tree , prove that $\nu = \epsilon - 1$
3. Prove that a connected graph is tree iff every edge is a cut edge.

UNIT-III

1. Define the closure of the graph G and prove that it is well defined.
2. Let G be a non-trivial simple graph with degree sequence $(d_1, d_2, \ldots d_\nu)$ where $d_1 \leq d_2 \leq d_3 \ldots \leq d_\nu$. show that if there is no value of m less than (v+1)/2 for which d_m<m and $d_{\nu+m+1}$< v-m then G has Hamiltonian path.
3. If G is non – Hamiltonian simple graph with v ≥ 3, then prove that G is degree - majorized by some $c_{m.\nu}$
4. Prove that c(G) is well defined.

UNIT-IV

1. Let M and N be disjoint matching of G with |M|>|N|. Then prove that there are disjoint matching M` and N` of G such that |M ′|=|M|-1, |N′|=|N|+1 and M′U N′=MUN.

2. Prove that α+β=v.
3. Prove that a set S⊆V is an independent set of G if and only if V\S is a covering set of G.

UNIT-V

1. If G is a k-critical graph with a 2-vertex cut (u, v) then prove that
   (i) G = G_1UG_2 where $G_1$is a {u, v} component of type I (i=1,2).
   (ii) Both G_1+uv and G_2.uv are k-critical.
2. If G is 4-chromatic then prove that G contain a subdivision of k_4
3. Prove that a graph is planar if and only if it contains no subdivision of k_5 or $K_{3,3}$
4. Obtain the chromatic polynomial of the following graph.

10 MARKS

UNIT-I

1. Prove that if G is simple $\delta[G] \geq \frac{v-1}{2}$ then g is connected.
2. State if G is simple and $\delta \geq 2$then G contains a cycle of length a atleast $\delta +1$
3. Prove that a graph is bipartite iff it contains no odd cycles.
4. State and prove sperner's lemma.
5. Explain the shortest path problem

UNIT-II

1. State and prove the cayley's formula

2. State and prove the whitney 's theorem
3. Prove that a vertex v of G is a cut vertex of G iff d(v)>1
4. If e is a link of G , prove that τ(G)= τ(G-e) +τ(G.e)
5. Prove that an edge 'e' of G is a cut edge of G iff 'e' contained in no cycle of G
6. Prove that a graph G with v≥3 is 2-connected iff any two vertices of G are connected by atleast two internally disjoint paths.
7. Prove that τ(K_n)=n^{n-2}
8. Prove k k' δ

UNIT-III

1. State and prove Chvatal theorem on Hamiltonian graph.
2. State Fleury's algorithm and Prove that is G is Eulerian then any trial in G constructed by Fleury's algorithm is an Euler tour of G.
3. Prove that c(G) is well defined.
4. Prove that if G is a simple graph with v≥3 and ε>($\frac{v+2}{2}$)+1 then G is Hamiltonian.
5. Prove that a non-empty connected graph is Eulerian if it has no vertices of odd degree.
6. Explain travelling salesman problem.
7. If G is simple graph with v≥3 and δ`≥ v/2 then prove that G is Hamiltonian.

UNIT-IV

1. If G is simple, then prove that either χ`=Δ or χ`=Δ+1.
2. Prove that r(k,k)≥$2^{k/2}$.
3. Prove that α`+β`=v.

4. State and prove that Veizing's Theorem.
5. For any two integers k≥2 and l≥2. Show that r(k,l)≤r(k,l-1)+r(k-1,l) further prove that strict inequality in the above holds if r(k,l-1) and r(k-1,l) are both even.

UNIT-V

1.State and Prove that Brooks' theorem.

2. Prove that k_5 is non-planar.

3.If G is a connected simple graph and is neither an odd cycle nor a complete graph ,then $\chi \leq \Delta$.

4. If G is 4- chromatic,then G contains a subdivision of K_4
5. If G is simple , then π_k(G)= π_k(G-e) - π_k(G.e)