St. Joseph's College of Arts and Science, Cuddalore.

Question Bank

PG Research Department of Mathematics

Class: I B.Sc Mathematics

Subject Name: Calculus

Subject Code: MT203S

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Calculus - 1st year, Semester II

Unit-1

2 MARKS

1. If x = rcosθ: y = rsinθ find $\frac{\partial(x,y)}{\partial(r,\theta)}$.
2. If u is the function of three independent variable x,y and z then write down the total differential of u.
3. If $xy = c^2$ find $\frac{dy}{dx}$.
4. If u= x+y; v= x-y find $\frac{\partial(x,y)}{\partial(u,v)}$.
5. State Leibnitz's Theorem.
6. If $y= \sin(m\sin^{-1}(x))$ Show that $(1-x^2)y_2 - xy_1 + m^2y = 0$.
7. Define saddle point.
8. Find the Liebnitz formula for the n^{th} derivative of a product of two functions u and v.

5 MARKS

1. Find $\frac{du}{dt}$ where $u = x^2+ y^2+ z^2$, $x = e^t$, $y = e^t\sin t$ and $z=e^t\cos t$.
2. If x = rcosθ: y = rsinθ then prove $\frac{\partial(x,y)}{\partial(r,\theta)}.\frac{\partial(r,\theta)}{\partial(x,y)} = 1$.
3. Find the minimum value of $x^2 + y^2 + z^2$ where ax + by + cz = p.
4. If z=f(x,y) where x = rcosθ and y = rsinθ show that

$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2.$

5. Find the n^{th} derivative of $e^{nx}\cos^2 x \sin x$.
6. Find yn, where $y=\frac{3}{(x+1)(2x-1)}$.

10 MARKS

1. If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$ and $z = r\cos\theta$ prove $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)} = r^2 \sin\theta$.
2. If $u = x+ y+ z$, $uv = y + z$, $uvw = z$ find $\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2v$
3. Find the maximum or minimum values of $f(x,y)= 2[x^2 - y^2] - x^4 + y^4$.
4. A rectangular box without a lid is to be made form12m^2 of cardboard. Find the volume of such a box.
5. Find the maxima and minima of $f(x,y)= x^3+ y^3- 3axy$.
6. If $y = (x +\sqrt{1 + x2}\,)^2$ Prove that $(1+x_2)Y_{n+2} + (2n+1)xy_{n+1} +(n^2-m^2)y_n =0$.
7. If $x=v^2 +w^2$, $y = w^2 + u^2$, $z = u^2 +v^2$ show that $\frac{\partial(x,y,z)}{\partial(u,v,w)}.\frac{\partial(u,v,w)}{\partial(x,y,z)} = 1$.
8. Find y_n when $y = \frac{x2}{(x-1)2(x+2)}$.
9. Discusss the maxima and minima of the function $x^3y^2(6-x-y)$.

UNIT 2

2 marks

1. Write the Cartesian formula for radius of curvature.
2. Find the radius of curvature of curve $y = e^x$ at the point where it crosses the y-axis.
3. Write the formulae for the radius of curvature for polar coordinates and for pedal curve.
4. Define chord of curvature.
5. Define radius of curvature`

6. Write the formula for radius of curvature in polar form.

5 marks

1. Find the curvature at any point of the curve
   $x=ae(\sin\boldsymbol{\theta} - \cos\boldsymbol{\theta})$, $y=ae^{\theta}(\sin\boldsymbol{\theta} + \cos\boldsymbol{\theta})$.
2. Find the (p-r) equation for the curve $r\sin\boldsymbol{\theta} + a = 0$.
3. Find the radius of curvature for the curve xy=30 at the point (3, 10).
4. Find the shape of the straight line $\frac{1}{r} = \cos(\boldsymbol{\theta} - \boldsymbol{\alpha}) + e\cos\boldsymbol{\theta}$.
5. What is the radius of curvature of the curve $x^4 + y^4 = 2$ at the point (1,1).

10 marks

1. Show that the ROC of the curve $y^2 = \frac{a2(a-x)}{x}$ at (a,0) is $\frac{1}{2}a$.
2. Show that in the parabola $y^2=4ax$ at the point t, $\boldsymbol{\rho} = -2a(1+t^2)^{3/2}$; Find the
3. Find the radius of curvature of $y^2 = a^3 - x^3 / x$ at (a,0).
4. Derive the formula for radius of curvature in polar coordinates.
5. Find '$\boldsymbol{\rho}$' at the point t of the curve x=a(cost +tsint), y=a(sint-tcost).
6. Prove that the (p-r) equation of the cardioid $r=a(1 - \cos\boldsymbol{\theta})$ is $p^2 = r^2 /2a$
7. Find the pedal equation for the curves i) $r=a(1-\cos\boldsymbol{\theta})$ ii) $\frac{1}{r} = 1+e\cos\boldsymbol{\theta}$.
8. Find the radius of curvature of the centroid $r = a(1+\cos\boldsymbol{\theta})$ at the point $(r,\boldsymbol{\theta})$.
9. Show that the radius of curvature of the curve

   $r^n = a^n\cos n\boldsymbol{\theta}$ is $a^n r^{-n+1}/n+1$.
10. Find the p-r equation of $r=a\sin\boldsymbol{\theta}$.

11. Find the radius of curvature at any point 'θ' on the curve $x=a(\theta+\sin\theta)$, $y=a(1-\cos\theta)$.

12. Find the radius of curvature at $\left(\frac{3a}{2},\frac{3a}{2}\right)$ to the curve $x^2+y^2=3axy$.

UNIT 3

2 marks

1. Find the p-r equation of $r=a\sin\theta$
2. Define Envelope of a curve.
3. Define evolute.
4. Find the reduction formula for $I_n=\int x^n e^{ax}dx$.
5. Define the envelope of the family.
6. What is an asymptote?
7. Find the envelope of the family of straight lines $x\cos\alpha + y\sin\alpha = p$, being the parameter.

5 marks

1. Find the envelope of the family of straight line $y = mx+\frac{a}{m}$, the parameter being t.
2. Find the envelope of the family of the curves $(x-\alpha)^2 + y^2 = 4\alpha$, where α is a parameter.
3. Fine the envelope of the family of circles $(x-a)^2 + y^2 = 2a$, where a is the parameter.
4. Find all the asymptotes of $(x^2/a^2)-(y^2/b^2) = 1$.
5. Find the envelope of $y= mx +\sqrt{a^2m2+b^2}$ where 'm' is a parameter.
6. Find the asymptotes of $x^3+2x^3y-4xy^2-8y^3-4x+8y = 1$.

10 marks

1. Find the asymptotes of the curve, $x^3+3x^2y-xy^2-3y^3+x^2-2xy+3y^2+4x+7 = 0$.
2. Find the evolute of the rectangle hyperbola $xy = c^2$.
3. Find the evolute of the ellipse $(x^2/a^2) + (y^2/b^2)=1$.
4. Find the rectilinear asymptotes of $2x^4-5x^2y^2+3y^4+4x^3-6y^3+x^2+y^2-2xy+1 = 0$.

5. Find the asymptotes of $x^3+2x^2y-xy^2-2y^3+4y^2+2xy+y+1 = 0$.
6. Show that the evolute of the cycloid $x=a(\theta-\sin\theta)$, $y=a(1-\cos\theta)$ is another cycloid.
7. Find the evolutes of asymptotes of the curve $y^3-x^2y-2xy^2+2x^3-7xy+3y^2+2x^2+2x+2y+1 = 0$.
8. Find the envelope of the ellipses having the axes of coordinates as principal axes and sum of their semi axes constant.
9. Find the envelope of $x/a + y/b =1$ subject to $a^n+b^n=c^n$ given C is a known constant.
10. Find the asymptotes of the cubic curve $y^3-6xy^2+11x^2y-6x^3+x+y = 0$.

UNIT 4

2 marks

1. Evaluate $\int_0^1 x7(1-x)^8 dx$ Using Gamma Beta function.
2. Define Gamma function.
3. Write down the I_n for $\int x^n e^{ax}$ using reduction formula.
4. Evaluate $\int x^2 e^{-2x} dx$.
5. Define Beta function.
6. Show that $(n+1) = n(n) = n!$
7. Prove that $\Gamma 1 = 1$.

5 marks

1.Evaluate $\int_0^{\pi/2} \sin^m x \cos^n x dx$

2. Establish a reduction formula for $\int \sin^n x dx$.

3. Evaluate $\int_0^{\pi/2} x(1-x^2)^{1/2} dx$

4. Find the reduction formula for $\int \cot^n x dx$ with limits $\pi/4$ to $\pi/2$.

5. Establish a reduction formula for $\int \cos^n x dx$.

6. Prove that $\Gamma\frac{1}{2} = \sqrt{\pi}$.

7. Evaluate $\int \sec^3 x dx$.

10 marks

1.Establish the relation between beta and gamma function.

2. Prove that (m,n) = $\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.

3.Evaluate $\int_0^{\pi/2}$ e^{-x2}dx.

4.Derive reduction formula for $I_{m,n}$ = $\int x^m(\log x)^n dx$; m,n>0 and hence deduce $\int x^4(\log x)^3 dx$.

5.Prove that $\Gamma(n+\frac{1}{2}) = \frac{\sqrt{\pi}}{2n}(2n-1)!$

6. If I_n= $\int_0^{\pi/2}$ $x\cos^n x dx$ where n>1 show that $I_n =(-1/n^2) +((n-1)/n)I_{n-2}$ Find I_4

7. Find the reduction formula for $\int_0^{\pi/2}$ $\sin^n x dx$ and hence evaluate $\int_0^{\pi/2}$ $\sin^8 x dx$

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8. If I_n = $\int_0^1$ $x^p(1-x^q)^n dx$ where p,q>0 and n is a positive integer, Prove that $(p+qn+1)I_n =nqI_{n-1}$ and hence evaluate I_3

UNIT 5

2 marks

1. Evoluate $\int_0^1 \int_0^2 (x^2+y^2)\, dydx$
2. Evoluate $\int_0^1 \int_0^1 dxdy$
3. How we can express the co-ordinates of the centre of gravity.
4. Prove that (m,n) = $\boldsymbol{\beta}$(n,m).
5. Evaluate $\int_0^h \int_0^h xy\, dx\, dy$

5 marks

1. Find the volume of a sphere of radius 'a' by triple integral.
2. Evoluate $\int\int xydxdy$ taken over the positive quadrant of the circle $x^2+y^2 = a^2$.
3. Find the area enclosed by the ellipse $(x^2/a^2) + (y^2+b^2) = 1$.
4. Evaluate $\int_0^1 (xlogx)^4dx$
5. Evaluate $\int_0^{\pi/2} \int_0^{acos\theta} r\sqrt{a^2-r^2}dr\, d\boldsymbol{\theta}$
6. Evaluate $\int_0^{a} \int_0^{\sqrt{ay}} xydydx$

10 marks

1. Find the area of the ellipse $x^2+4y^2-6x+8y+9 = 0$.
2. Find the volume of the tetrahedrom bounded by the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c} = 1$ and the coordinate planes.
3. Evaluate $\int\int\int xyz\, dxdydz$ taken through the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$.
4. Evaluate $\int_0^1 \int_0^1 \int_0^1 (x+y+z)\, dx\, dy\, dz$
5. Find the area common to $y^2 = 4ax$, $x^2 = 4ay$
6. Evaluate $\int_0^{log2} \int_0^{x} \int_0^{x+logy} e^{x+y+z}dzdydx$
7. Evaluate $\int_0^4 \int_0^{x} \int_0^{\sqrt{x}+y} z\, dz\, dy\, dx$
8. Evaluate $\int_{-a}^{a} \int_0^{\sqrt{a2-y2}} x\, dx\, dy$ change the order of integration.
9. Evaluate $\int_0^{\infty} \int_x^{\infty} \frac{e^{-y}}{y} dy\, dx$ change the order of integration.