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SUBJECT: MECHANICS-I **SUBJECT CODE:** MT407

QUESTION BANK

UNIT – I: FORCE

2– Mark Questions:

1. Define force.
2. Find the expression for magnitude of the resultant $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$.
3. What is the resultant of two forces $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$.
4. State triangle law of forces.
5. State the Converse of triangle of forces.
6. Express the force $\vec{F}$ acting in a plane in terms of its components in two perpendicular directions in the plane.
7. State Lami`s theorem.
8. ABC is a triangle G is its centroid and P is any point in the plane of the triangle. Show that the resultant of forces represented by $\overrightarrow{PA}$,$\overrightarrow{PB}$,$\overrightarrow{PC}$ is 3$\overrightarrow{PG}$.
9. Define coplanar motion.
10. If the resultant of the forces 3p, 5p is equal to 7p. Find the angle between the forces.
11. If the two forces $\vec{P}$,$\vec{Q}$ acting at a point is such that their sum and difference are perpendicular to each other, show that P=Q.
12. What is rectilinear motion?

5 – Mark Questions:

1. State and prove Lami's theorem.
2. Forces3, 2, 4, 5kg weight act along the sides AB, BC, CD and DA of a square. Find their resultant and its line of action.
3. A weight is supported on a smooth plane of inclination α by a string inclined to the horizontal at angleγ. If the slope of the plane be increased to β and the slope of the string be unaltered, the tension of the string is doubled. Prove that cot α - 2cot β=tan γ.
4. The magnitude of the resultant of two given forces P, Q is R. If Q is doubled then R is doubled. If Q is reversed then also R is doubled. Show that P: Q: R = $\sqrt{2} : \sqrt{3} : \sqrt{2}$.

5. State and prove triangle law of forces, also prove the converse.
6. Three forces acting at a point are parallel to the sides of the triangle ABC taken in order and in magnitude they are proportional to the cosines of the opposite angles. Show that the magnitude of their resultant is proportional to $\sqrt{1-8\cos A\cos B\cos C}$.

7. The magnitude of the resultant of the forces $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ on a particle is equal to the magnitude of $\overrightarrow{F_1}$. Where the first force is doubled, show that the new resultant is perpendicular to $\overrightarrow{F_2}$.

8. 'O' is the orthocentre of the triangle ABC. If forces of magnitude P, Q, R acting along OA, OB, OC are in equilibrium. Show that $\frac{P}{a} = \frac{Q}{b} = \frac{R}{c}$.

9. Three like parallel forces P, Q, R act at the vertices of a triangle ABC. If their resultant passes through the orthocentre O, Show that $\frac{P}{\tan A} = \frac{Q}{\tan B} = \frac{R}{\tan C}$.

10. The resultant of two forces P, Q is magnitude P. Show that if P is doubled, the new resultant is perpendicular to the force Q and its magnitude is $\sqrt{4P^2 - Q^2}$.

11. `I` is the incentre of the triangle ABC. If forces of magnitudes P, Q, R acting along the bisectors IA, IB, and IC are in equilibrium, show that $\frac{P}{Cos\left(\frac{A}{2}\right)} = \frac{Q}{Cos\left(\frac{B}{2}\right)} = \frac{R}{Cos\left(\frac{C}{2}\right)}$.

12. A uniform plank AB of length 2a and weight 'w' is supported horizontally on two horizontal pegs 'C' and 'D' at a distance 'd' apart. The greatest weight that can be placed at the two ends in succession without upsetting the plank are w_1 and w_2 respectively. Show that $\frac{w}{w+w_1} + \frac{w}{w+w_2} = \frac{d}{a}$.

10– Mark Questions:

1. The magnitude of the resultant of two given forces P,Q is R. If Q is doubled, then R is doubled. If Q is reversed then also R is doubled. Show that P : Q : R $= \sqrt{2} : \sqrt{3} : \sqrt{2}$

2. The resultant of two forces of magnitudes P and Q acting on a point has magnitude $(2n+1)\sqrt{P^2+Q^2}$ and $(2n-1)\sqrt{P^2+Q^2}$ when the forces are inclined at α and $90^0 - \alpha$ respectively. Show that $\tan\alpha = \frac{n-1}{n+1}$.

3. State the triangle law of forces and prove its converse.

4. State and prove the Lami's theorem.

5. The magnitude of the resultant of two forces P,Q is R. If P is doubled, then R is doubled. If Q is doubled and reversed, then also R is doubled. Show that $P:Q:R = \sqrt{6}:\sqrt{2}:\sqrt{5}$.

6. A weight is supported on a smooth plane of inclination α by a string inclined to the horizon at an angle γ . If the slope of the plane be increased to β and the slope of the string unaltered, the tension of the string is doubled. Prove that $\cot\alpha - 2\cot\beta = \tan\gamma$.

7. S is the circum centre of a triangle ABC. If forces of magnitude P, Q, R acting along

   SA, SB, SC are in equilibrium, Show that P,Q,R is in ratio

   (i) $\dfrac{P}{Sin2A} = \dfrac{Q}{Sin2B} = \dfrac{R}{Sin2C}$

   (ii) $\dfrac{P}{a^2(b^2+c^2-a^2)} = \dfrac{Q}{b^2(c^2+a^2-b^2)} = \dfrac{R}{c^2(a^2+b^2-c^2)}$

8. ABC is a given triangle . Force P, Q , R acting along the lines OA, OB, OC are in equilibrium. Prove that

a) $\dfrac{P}{\cos\frac{A}{2}} = \dfrac{Q}{\cos\frac{B}{2}} = \dfrac{R}{\cos\frac{C}{2}}$ if O is the incentre of the triangle.

b) $\dfrac{P}{a} = \dfrac{Q}{b} = \dfrac{R}{c}$ if O is the orthocentre of the triangle.

9. Find the magnitude and direction of the resultant of $\vec{F_1}$ and $\vec{F_2}$.

10. Two forces of magnitude F_1 and F_2 act at a point. They are inclined at an angle α. If the forces are interchanged show that the resultant is turned through the angle

    $2\tan^{-1}\left(\dfrac{F_1-F_2}{F_1+F_2}\tan\dfrac{\alpha}{2}\right)$

11. S and H are the circum center and orthocenter of a triangle ABC. Show that

    (i) The resultant of the forces $\bar{SA}, \bar{SB}, \bar{SC}$ acting at S is $\overline{SH}$.

    (ii) The resultant of the forces $\overline{HA}$, $\overline{HB}$, $\overline{HC}$ acting at H is $2\overline{HS}$

12. A rod of length 5a is bent so as to form 5 sides of a regular hexagon. Show that its centre of mass is at a distance $a\sqrt{1.33}$ from either end of the rod.

13. Weights W, w, W are attached to points B, C, D respectively of a light string AE, Where B, C , D divide the string into 4 equal lengths. If the string hangs in the form of 4

consecutive sides of a regular octagon with the ends A and E attached to point on the same level, Show that W = ($\sqrt{2}$ + 1)w.

14. A uniform plank AB of length '2a' and weight W is supported horizontally on two horizontal pegs C and D at distance'd' apart. The greatest weights that can be placed at the two ends in succession without upsetting the plank are W1 and W2 respectively. Show that $\frac{W_1}{W+W_1} + \frac{W_2}{W+W_2} = \frac{d}{a}$

UNIT – II: EQUILIBRIUM OF A PARTICLE

2– Mark Questions:

1. Define a couple.
2. Three like parallel forces P, Q, R act at the vertices of a triangle ABC. Show that their resultant passes through the centroid if P=Q=R.
3. Define Moment of a force.
4. Forces of magnitudes 3p, 4p, 5p act along the sides BC, CA, AB, of a equilateral triangle of side a. Find the moment of the resultant about A.
5. Two like parallel forces of magnitudes P, Q act on a rigid body. If Q is changed to P^2/Q, with the line of action being the same, show that the line of action of the resultant will be the same as it would be, if the forces were simply interchanged.
6. Define moment of a force about a line.
7. Define the arm and axis of a couple.
8. What do you say that a particle is in equilibrium?
9. Define moment of a couple.

5 – Mark Questions:

1. Show that if a particle under several forces is in equilibrium, the sums of the components of the forces in two mutually perpendicular directions.

2. A particle C of weight W is in equilibrium being supported by two strings CA, CB of length 4a, 3a respectively and acted on by a horizontal force W is in the plane ABC. If the ends A, B are at the same level and at a distance 5a apart, show that the tensions in the strings are 60^c.

3. Two like parallel forces of magnitudes P, Q act on a body. If the second force is moved away from the first paralleled through a distance d, show that the resultant of the force moves through a distance $\frac{dQ}{P+Q}$.

4. A particle C of weight W is in equilibrium being supported by two strings CA, CB of length 4a, 3a respectively and acted on by a horizontal force W in the plane ABC of the ends A, B are at the same level and at a distance 5a apart, show that the tensions in the strings are 7W/5, W/5.

5. Three forces acting along the sides of a triangle in the same order are equivalent to a couple. Show that they are proportional to the sides of the triangle.

6. If two like parallel forces of magnitudes P, Q ($P > Q$) acting on a rigid body at A, B are interchanged in position, show that the line of action of the resultant is displaced through a distance $\frac{AB(P-Q)}{P+Q}$.

10 – Mark Questions:

1. Let E is the midpoint of the side CD of a square ABCD. Forces 16, 20, $4\sqrt{5}, 12\sqrt{2}$ act along $\overrightarrow{AB}, \overrightarrow{AD}, \overrightarrow{EA}, \overrightarrow{CA}$. Show that they are in equilibrium.

2. Find the resultant of two parallel forces acting on a rigid body.

3. State and prove the Varignon's theorem.

4. Five forces acting at a point are represented and the direction by the lines joining the vertices of any Pentagon to the mid points of their opposite sides .Show that they are in equilibrium.

5. ABCDEF is a regular hexagon. Forces P, 2P, 3P, 2P, 5P, 6P act along AB, BC, DC, ED, EF, and AF. Show that the six forces are equivalent to a couple and find the moment of the couple.

6. Three forces acting along the sides of a triangle in the same order are equivalent to a couple. Show that the forces are proportional to the sides of a triangle.

7. Three like parallel forces P,Q,R act at the vertices of a ΔABC.Show that their resultant passes through ,
   (i) The centroid if P = Q = R,
   (ii) The centroid if $\frac{P}{a} = \frac{Q}{b} = \frac{R}{c}$.

UNIT – III: FRICTION

2– Mark Questions:

1. Define the coefficient of friction.
2. Define angle of friction.
3. Write the laws of static friction.

4. Write the laws of friction.
5. Write the laws of dynamical friction.
6. Define cone of friction.
7. Show that the scented inclination of a regular inclined plane to the horizon so that a partied remains on it at rest, is equal to the angle of friction.
8. Write a note on friction.
9. Define angle and cone of friction.

5 – Mark Questions:

1. State the laws of friction.

2. Show that the greatest inclination of a rough inclined plane to the horizon, so that a particle will remain on it at rest, is equal to the angle of friction.

3. Find the least force required to drag particle on a rough horizontal plane and show that the least force acts in a direction making with the horizontal, an angle of friction.

4. A particle is placed on a rough plane inclined at an angle α to the horizontal. If the force, which acting parallel to the plane in the upward sense, is just sufficient to keep the particle at the point of moving up the plane, is n times the force, which acting in the same manner, is just sufficient to keep the particle at the point of moving down the plane, show that

   $\tan \alpha = \mu \dfrac{n+1}{n-1}$, where μ is the coefficient of friction.
5. A particle rests on a plane inclined at 45^{c} to the horizontal, being supported by a string along the line of the greatest slope. If the ratio of the maximum and minimum tensions consistent with equilibrium is 2:1, find the coefficient of friction.

6. A partied rest S on a plane inclined at 45^{c} to the horizontal being supported by a string along the line of the greatest slope. If the ratio of the maximum and minimum tensions consistent with equilibrium is 2:1, find the coefficient of friction.

7. A weight is to be transported from the bottom to the top of an inclined plane whose inclination to the horizontal is α. Show that a smaller force will be required to drag it along the plane than to left it, provided the coefficient of friction is less than tan $(\frac{\pi}{4} - \frac{\alpha}{2})$.

10 – Mark Questions:

1. Explain about the laws of friction.

2. Suppose a particle of mass m is placed on a rough inclined plane inclined at an angle α to the horizontal and a force of magnitude S acts on it in a direction making an angle θ with the plane. If the equilibrium is limiting, Find S.

3. Two rough particles of masses m_1 and m_2 connected by a light string rest on an inclined plane with the string lying alone a line of greatest slope. If the particles are in limiting equilibrium, Show that $\tan\alpha = \frac{\mu_1 m_1 + \mu_2 m_2}{m_1 + m_2}$, Where α is the inclination of the plane and μ_1, μ_2 are the coefficients of friction of the particles.

4. A weight is to be transported from the bottom to the top of an inclined plane whose inclination to the horizontal is α Show that a smaller force will be required to drag it along the plane than to lift it, provided the coefficient of friction is less than $\tan\left(\frac{\pi}{4} - \frac{\alpha}{2}\right)$.

5. Two equally rough inclined planes inclined at angles α and $\beta, (\alpha > \beta)$, to the horizon, have a common vertex. A string passing over a small smooth pulley at the vertex supports two particles of equal mass. If the particles are in the limiting equilibrium, show that μ, the coefficient of friction is $\tan\left(\frac{\alpha - \beta}{2}\right)$.

UNIT – IV: CENTRE OF MASS

2– Mark Questions:

1. Find the mass centre of the three particles of certain masses.
2. Where will be the centre of gravity of a triangular area?
3. Define centre of mass.
4. Find the mass centre of the lamina in the form of a quadrant of a circle.
5. Find the mass centre of a triangular lamina.
6. Define centre of gravity of a body.
7. Suppose the position of three given particles as A, B, C and their masses are proportional to their length. Then find the mass centre.
8. Write down the formula for position rector of mass centre wring integration.
9. What is the position of mass centre of a hollow right circular cone?
10. What is the mass center of a solid hemisphere of radius $\propto$.

5 – Mark Questions:

1. Form the circular lamina of radius α, circular portion of radius $\frac{1}{2}\alpha$ is so stamped out that its centre bisects a radius of the given lamina. Find the centre of gravity of remaining portion.

2. A triangle lamina ABC right angled at A is suspended from A and AB=3AC. Show that, if the hypotenuse is inclined to the vertical at an angle θ then $\sin\theta = \frac{3}{5}$.

3. A rod of length 5a is bent so as to form five sides of a regular hexagon. Show that its centre of mass is at a distance a $\sqrt{1.33}$ from either end of the rod.

4. From the circular lamina of radius a, a circular portion of radius $\frac{a}{2}$ is so stamped out that its centre bisects a radius of the given lamina. Find the centre of mass of the remaining portion.

5. Find the mass centre of a solid hemisphere of radius `a`.

6. Find the mass centre of a thin wire in the form of a circular arc.

7. OA and OB are two uniform rods of lengths 2a, 2b. If angle AOB=α, show that the distance of the mass centre of the rods from O is $\frac{(a^4+2a^2b^2\cos\alpha+b^4)^{\frac{1}{2}}}{a+b}$.

8. From a solid cylinder of height H, A cone whose base coincides with the base of the cylinder is scoped out so that the mass centre of the remaining solid coincides with the vertex of the cone. Find the height of the cone.

9. Show that the mass centre of a triangle frame is the incentre of the triangle.

10. A square hole is punched out of a circular lamina of radius ‘a’, having a radius as its diagonal. Show that the distance of C.G. of the remainder from the centre of the circle is $\frac{a}{4\pi-2}$.

10 – Mark Questions:

1. Find the mass centre of the lamina in the form of quadrant of an ellipse of axes 2a, 2b.

2. Find the mass centre of a solid right circular cone of height h.

3. Find the mass centre of a solid hemisphere of radius a.

4. Find the centre of Cardioidal lamina.

5. Find the mass centre of a solid right circular cone of height h.

6. To find the mass centre of the thin wire in the form of circular arc.

7. If the mass centre of a quadrilateral lamina is the point of intersection of the diagonals. Show that the quadrilateral is parallelogram.

8. A square hole is punched out of a circular lamina of radius ‘a’, having a radius as its diagonal. Show that the distance of centre of gravity of the remainder from the centre of the circle is $\frac{a}{4\pi-2}$.

9. A triangular lamina ABC right angled at A is suspended from A and AB = 3AC. Show that, if the hypotenuse is inclined to the vertical at an angle θ then $\sin\theta = \frac{3}{5}$.

10. D,E,F are the midpoints of the sides BC,CA,AB of a ΔABC. Masses m_1, m_2, m_3 are placed at A,B,C and masses M_1, M_2, M_3 are placed at D,E,F.If the two systems have the same mass centre, show that,
$\frac{m_1}{M_2+M_3} = \frac{m_2}{M_3+M_1} = \frac{m_3}{M_1+M_2}$.

11. To find the centre of gravity of three uniform rods forming a triangle.

UNIT – V: KINEMATICS

2– Mark Questions:

1. Define kinetic energy.
2. A train travels at the rate of 27km/h. Rain which is falling vertically appears to a man in the train to make an angle of $\tan^{-1} 6/5$ with the vertical. Find the magnitude of the true velocity of the rain.
3. What do mean by relative velocity?
4. A ship steaming north at $8\sqrt{3}$ km.p.h. and a man walks across its deck in a direction due west at 8 km. per hour. Find his resultant velocity in space.
5. Define the angular velocity.
6. A particle moves along straight line with constant acceleration. Show that v=u+at.
7. A boat which can steam is still water with a velocity of 48 km. p.h is steaming with its slow pointed due east when it is carried by a current which flow northward with a speed of 14 km. p.h. Find the actual distance it would travel in 12 minutes.
8. Define relative velocity.
9. Distinguish between velocity and speed.
10. In the radial and transverse direction components, if the path of the particle is a circle with at O and radius is a, what will be the velocity.

5– Mark Questions:

1. A string ABCD hangs from fixed points A, D carrying a mass of 12kg at B and a mass of m kg at C.AB is inclined at 60^c to the horizontal, BC is horizontal and CD is inclined at 30^c to the horizontal. Show that m=4.

2. A ship sails northeast at 15 km. p. h and to a passenger on board, the wind appears to blow from north with a velocity of $15\sqrt{2}$ km. p. h. Find the true velocity of the wind.

3. A stream is running at a speed of 4ml.p.h. Its breadth is one quarter of a mile. A man can row a boat at a speed of 5ml.p.h. In still water. Find the direction in which he must row in order to go perpendicular to the stream and the time it takes him to cross the stream so.

4. Show that the angular velocity about a fixed point A of a particle P moving uniformly in a straight line varies inversely as the square of the distance of the line from the fixed point.

5. A man seated in a train whose velocity is 80 Km per hour throes a ball horizontally and perpendicular to the train with a velocity of 60 Km per hour. Find the velocity of the ball immediately after the throes.

6. A man can swim perpendicularly across a stream of breadth 100m in 4 minutes when there is on current and in 5 minutes when there is a downward current. Find the velocity of the current.

7. A body of weight 4kg rest in limiting equilibrium on a rough plane whose slope is 30^c. If the plane is raised to a slope of 60^c, find the force along the plane required to support the body.

8. If a point moves in a straight line with uniform acceleration and covers successive equal distances in times t_1, t_2, t_3 then show that $\frac{1}{t_1} - \frac{1}{t_2} + \frac{1}{t_3} = \frac{3}{t_1+t_2+t_3}$.

9. Two particles A, B describe a circle of radius ‘a’ in the same sense and with same speed ‘u’. Show that the relative angular velocity of each with respect to the other is $\frac{u}{a}$.

10. Let a particle move along a straight line with a constant acceleration ‘a’. With the usual notations, show that v=u+at, s=ut + $\frac{1}{2}at^2$ and $v^2 = u^2 + 2as.$

11. A train travels at the rate of 27km/hr. Rain which is falling vertically appears to a man in the train to make an angle of $\tan^{-1} 6/5$ with the vertical. Find the magnitude of the true velocity of the rain.

10 – Mark Questions:

1. Find the components of the acceleration of a particle in the tangential and normal directions.

2. The speed of the train increases at a constant rate α from 0 to v, and then remains constant for an interval and finally decreases to 0 at a constant rate β. If s is the total distance describe, prove that the total time T occupied is T=$\frac{s}{v}+\frac{v}{2s}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)$.

3. The line joining two points A, B is of constant length a and the velocities of A, B are in directions which make angles α and β respectively with AB. Prove that the angular velocity of AB about A is $\frac{u \sin(\beta-\alpha)}{a \cos \beta}$, where u is the velocity of A.

4. Find the components of velocity and acceleration of a particle in the radial and transverse direction.

5. For a man walking along a level road at 5km/h the rain appears to beat into his face at 8km/h at an angle 60^c with the vertical. Find the direction and magnitude of the true velocity of the rain.

6. A vertical circular disc of radius rolls on a ground without slipping along a straight line with a linear velocity u. Find the velocity of any point on its rim.

7. Derive the equation of rectilinear motion of a part constant acceleration.
   Obtain the components of velocity and acceleration P moving in a plane along
   (i). Two fixed perpendicular direction.
   (ii). The radius and transverse direction.

8. i) If a point moves in a straight line with uniform acceleration and covers successive equal distances in times t_1, t_2, t_3 then show that $\frac{1}{t_1} - \frac{1}{t_2} + \frac{1}{t_3} = \frac{3}{t_1 + t_2 + t_3}$.
   ii) The two ends of a train moving with a constant acceleration pass a certain point with velocity u and v respectively. Show that the velocity with which the middle of the train passes the same point is $\sqrt{\frac{1}{2}\left(u^2 + v^2\right)}$.

9. Derive the equation of Rectilinear motion of a particle with constant acceleration.

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