# Department of Mathematics

# QUESTION BANK

## Class: II B.Sc Mathematics

## Sub Name: Graph theory

## Sub Code: MT408

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### Graph theory

### unit-1

**SECTION – A**

1. Define bipartite graph
2. State Whitney's theorem
3. Define multigraph with an example.
4. Prove that the sum of the degrees of the points of a graph G is twice the number of lines.

**SECTION – B**

1. Prove that any self complementary graphs has 4n or 4n+1 points
2. Prove that every cubic graph has an even number of points
3. Let G be a (p,q) graph. Prove that L(G) is a (q, $q_n$) graph where $q_L = \frac{1}{2} i \left( \sum_{i=1}^{p} d_i^2 \right) - 9$

**SECTION – C**

1. a) i) Prove that the sum of the degrees of the points of a graph G is twice the number of lines. ii) Prove that r (m,n) = r ( n , m ).

b) i)Prove that in any graph G the number of points of odd degree is even . ii) Prove that r (2,2 )= 2.

2. a) i) Prove that for any graph G with 6 points, G or $G$ contains a triangle.

   ii) Prove that $\gamma(m, n) = \gamma(n, m)$

3. i) Let G be a (p,q) graph all of whose points have degree k or k+1. If G has t>0 points of degree k, show

   that $t = p\,(k+1) - 2q$

   ii) Show that in any group of two or more people, there are always two with exactly

   the same number of friends inside the group.

4. Prove that the maximum number of lines among all p point graphs with no triangle is $\left[\dfrac{p^2}{4}\right]$ ($[x]$denote the greatest not exceeding the real number x)

5. i) Prove that $\delta \le \dfrac{2p}{p} \le \Delta$

   ii) Let G be a K-regular bigraph with bipartition ($v_1, v_2$) and k>0. Prove that $|v_1| = |v_2|$

# Unit-2

## SECTION – A

1. Define graphic sequence and realization of a graph G.

2.Show that the degree sequences partition P=(7,6,5,4,3,2,) is not graphic.

3. Define a trial and path of a connected graph G.

4. Define union of two graph with suitable example.

## SECTION – B

1.Show that the partition P = { 6,6,5,4,3,3,1 } is not graphic

2.Prove that in a graph G, any u-v walk contains a u-v path.

3. If A is adjacency matrix of a graph with V = $\{v_1, v_2, \ldots. v_n\}$, prove that for any n ≥ 1 the ( i, j

)th entry of $A^n$ is the number of $v_i - v_j$ walks of length n in G.

4.If a partition $(d_1, d_2, ...d_p)$ with $d_1 \geq d_2 \geq ... \geq d_p$ is graphics, prove that $\sum_{i=1}^{p} di$ is even and $\sum_{i=1}^{k} di \leq k(k-1) + \sum_{i=k+1}^{p} \min\{k, d_i\}$ for 1<k<p.

**SECTION – C**

1. i) If $\delta \geq k$, Prove that G has a path of length K.

ii) Prove that a closed walk of odd length contains a cycle

2. Let $G_1$ be a $(p_1, q_1)$ graph and $G_2$ be a $(p_2, q_2)$graph.

Prove that i) $G_1 + G_2$ is a $(p_1 + p_2, q_1 + q_2 + p_1 p_2)$ graph.

ii) $G_1$ X $G_2$ is a $(p_1 p_2, q_1 p_2 + q_2 p_1)$

3.Prove that a closed walk of odd length contains a cycle.

4. Show that A partittion $P = (d_1, d_2, d_3....d_p)$ an even number into p parts with $p - 1 \geq d_1 \geq d_2 \geq ... \geq d_p$ is graphical if the modified partition $P^1 = (d_2 - 1, d_3 - 1, ...d_{d1+1} - 1, ..., d_{d1+2}, ...d_p)$is graphical.

5. If A is the adjacency matrix of a graph with v= $\{v_1, v_2, ...v_p\}$ prove that for any n≥1 the (i,j)$^{th}$ entry of $A^n$ is the number of $v_i$-$v_j$ walk of length n in G.

6.Define the following with sutable examples.

i) Adjacency matrix ii) Incidence matrix iii) Product and iv) Composition

# unit-3

**SECTION – A**

1..Write the condition for a graph G is n- connected.

2..Define Block with example

3. Give an example of a block of a graph G.

4..Prove that if G is a K-connected graph then $q \geq pk_2$

**SECTION – B**

1..If a line x of a connected graph G is a bridge then prove that x is not on any cycle of G.

2.. Prove that A line $x$ of a connected graph G is a bridge iff $x$ is not on
any cycle of G.

3..Prove that Every non-trivial connected graphs has atleast two points which are not cut
points.

**SECTION – C**

1.. A graph G with at least two points is bipartite iff all its cycles are of even length.

2..A line x of a connected graph G is a bridge iff x is not on any cycle of G.

3.. Prove that a graph G is connected if for any partition of V into subsets $V_1$ &$V_2$ there is a line of G joining a point of $V_1$ to a point of $V_2$.

4..Prove that a graph G with atleast two points is bipartite if all its cycles are of even length

5.. For any graph $G$, prove that $k \leq \lambda \leq \delta$

6..Let v be a point of a connected graph G. Show that the following statements are equivalent.

i).v is a cut point of G.

ii).There exists a partition of $v = \{v\}$in to subsets U and W such
that for each $u \in U$ and $w \in W$ the point $v$ is on every $u - w$ path.

iii).There exist two points u &w distinct from $v$ such that $v$ is on every u-w path.

# Unit-4

1. Draw a that a graph with 6 vertices.
2. Define tree with example
3. What is Eulerian trial?
4. Prove that every hamiltonian graph is 2-connected.

## SECTION – B

1. If G is a graph in which the degree of every vertex is at least two then prove that G contains a cycle.

2. Prove that every tree has a centre consisting of either one point or two adjacent points.

3. Prove that every tree has a centre consisting of either one point or two adjacent points.
4. Let G be a graph with p points and let u and v be non adjacent points in G such that $d(u) + d(v) \geq p$.Prove that G is Hamiltonian if G+uv is Hamiltonian.

## SECTION – C

1. Prove that the following statements are equivalent.

   i).G is Eulerian

   ii).Every point of G has even degree.

   iii).The set of edges of G can be partitioned into cycles.

2..Let G be (p, q ) graph. Prove that the following statements are equivalent.

(a) G is a tree.
(b)Every two points of G are joined by a unique path.
(c)G is connected and p = q+1.
(d)G is acyclic and p = q + 1.

3.. Let G be a graph with degree sequence $(d_1,d_2,\ldots,d_p)$, where $d_1 \leq d_2 \leq \ldots \leq d_p$ & $p \geq 3$ .Suppose that for every value of m less than p/2, either $dm > m$ or $d_{p-m} \geq P - m$. then prove that G is Hamiltonian

4.. i) Prove that every 2-connected plane graph can be embedded in the plane so that any specified face is the exterior face

ii) Prove that every polyhedron has atleast two faces with the same number of edges on the boundary.

5. If G is a graph with p≥3 vertices &$\delta \geq p/2$ , prove that G is Hamiltonian
6. Prove that C(G) is well defined.

# Unit-5

## SECTION – A

1. Define colouring of a graph.
2. State Euler's theorem
3. Draw a planar graph for $K_5$
4. State Kuratowski's theorem

## SECTION – B

1. If G is a (p , q) plane graph in which every face is an n cycle then prove that $q = ^{n\,(p-2)}.$
   .
2. If G is a plane connected (p,q) graph without triangles and p≥3, prove that q ≤ 2p-4.
3. If a $(p_1, q_1)$ graph and a $(p_2, q_2)$ graph are isomorphic, then prove that $p_1 + q_2 = p_2 + q_1$
4. Prove that if is uniquely n-colorable, then $\delta(G) \geq n - 1$.

## SECTION – C

1. If G is connected plane graph having V, E, and F as the sets of vertices, edges and faces respectively, then $V - E + F = 2$ .
2. i) Prove that every 2-connected plane graph can be embedded in the plane so that any specified face is the exterior face
   ii) Prove that every polyhedron has atleast two faces with the same number of edges on the boundary
3. i) If G is a (p,q) plane graph in which every face is an n cycle
   prove that $q = \frac{n\,(p-2)}{n-2}$
   ii) In any connected plane (p,q) graph (p≥3) with r faces $q \geq \frac{3r}{2}$ *and* $q \leq 3p\text{-}6$
4. Prove that $K_{3,3}$ is non planar.
5. Prove that the following statements for any graph G.
   a) If G is 2 – colourable then G is bipartite.
   b) If G is bipartite then every cycle of G has even length.
6. Prove that $\chi'(k_n) = n$ if n is odd $(n \neq 1)$ and $\chi'(kn) = n - 1$if n is even.