St. Joseph's College of Arts & Science (Autonomous)
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PG Research Department of Mathematics

Class: I M.Sc mathematics

Subject Name: Algebra II

Subject Code: PMT806S

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ALGEBRA-II

UNIT-I

TWO MARK QUESTIONS

1. Define algebraic numbers.
2. What is degree of $\sqrt{2}+\sqrt{3}$ over Q the fieldof rational numbers? Justify.
3. Define monic polynomial.
4. Define exentsion field.

FIVE MARK QUESTIONS

1. If a,b is k are alogebric over F, then prove that a$\pm$b,ab and $\frac{a}{b}$ (b$\neq$0) are algebraic over F.
2. Prove that if L is a finite extension of K and if K is a finite extension of F then L is a finite extension of F and [L:F]=[L:K][K:F]
3. Find the length of the curve given as the intersection of the surface $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, x=cosh($\frac{z}{a}$), from the point (a,0,0) to the point (x,y,z).

TEN MARK QUESTIONS

1. If L is a finite extension of k and if k is a finite extension of F then prove that L is a finite extension of F.
2. Prove that the elements a$\in$k is algebraic over F iff F(a) is a finite extension of F.
3. Prove that F(a, b)=F(b, a)
4. If a, b in K are algebraic over F, prove that a$\pm$b, a b, $\frac{a}{b}$ (b$\neq$0) are algebraic over F.

UNIT-II

TWO MARK QUESTIONS

1. Define splitting field.
2. If a$\in$k is a root p(x) $\in$F[x] of where F $\subset$K, then in K [X] prove that (x-a)/p(x).
3. Define roots of polynomial.

FIVE MARKQUESTIONS

1. Prove that if f(x)$\in$F[x] of degree n$\geq$1 then there is an extension E of F of degree at most n! is which f(x) has n roots.
2. Prove that a polynomial of degree 'n' over a field can have at most in roots in any extension field.
3. Obtain the equation of the tangent at any point on the circular helix

TEN MARK QUESTIONS

1. Prove that any splitting fields E and E' of the polynomials f(x)$\in$F[t] respectively, are isomorphic ϕ with the property that
   $\alpha\ \phi = \alpha$' for every $\alpha \in$F.
2. Prove that polynomial of degree 'n' over a field can have at most n roots in any extension field.
3. If p(X) is irreducible in F[x] and if v is a root of p(x), then prove that F(v) is isomorphic to F'(w) where w is a root of p(t); moreover this isomorphic σ can so be chosen that
   i)vσ=w
   ii) $\alpha\ \sigma = \alpha$' for every $\alpha \in$F.
4. If p(x) is a polynomial in F[X] of degree n$\geq$1 and irreducible over F, then prove that there is an extension E of F, such that [E:F]=n, in which p(x) has a root.
5. If f(x)ϵF(x), then prove that there is a finite extension E of F in which f(x) has root
6. If aϵK is a root of p(x) ϵ f(x) where F$\subset$K, then prove that in K[x](x-a)p(x).

UNIT-III

TWO MARK QUESTIONS

1. Prove that the fixed field of G is a subfield of K.
2. If F is of characteristic O and f(x) ϵ F(x) is such that f'(x)=0, prove that f(X)=αϵF.
3. Define characteristic zero.
4. Define fixed field.

FIVE MARK QUESTIONS

1. If K is a finite extension of F, then prove that G (K, F) is a finite group then prove that O (G (K, F)) $\leq$ [K: F] .
2. Prove that K is a normal extension of F if and only if K is the splitting field of some polynomials over F.
3. For any f(x),g(x) $\in$ F[x] and for any αϵ F prove that
   i) [f(x)+g(x)]'=f'(x)+g'(x)
   ii) [a f(x)]'=a. f'(x)

TEN MARK QUESTIONS

1. If F is of characteristic O and if a, b are algebraic over F, then prove that there exists element C $\in$ F (a, b) such that F (a, b) = F(c).
2. State and prove that fundamentals theorem of Galois theory.
3. Prove that the polynomials f(x) $\in$ F[x] has a multiple root if and onli if f(x) and f'(x) have a nontrivial common factor.
4. For any f(x),g(x) $\in$ F[x] and for any αϵ F prove that
   i) [f(x)+g(x)]=f'(x)+g'(x)
   ii) [a f(x)]'=a. f'(x)
   iii) [f(x) g(x)]'= f'(x) g(x)+f(x) g'(x)

5. Let K be a normal extension F and H be a subgroup of G(K:F); let K_H={X$\in$K|;σ (x)=x for all $\sigma \in$H} be the fixed field of H. prove that
   i) [K:K_H]=O(H);
   ii) H=G(K,K_H)
6. If K is a finite extension of F, then prove that G (K, F) is a finite group and its O(G(K,F)) satisfies O(G(K,F))$\leq$ [K:F].
7. Prove that K is a normal extension of F if and only if K is the splitting field of some polynomials over F.
8. If F is a field characteristic p≠0 , then prove that the polynomials xxp^n - x∈ F(x), for n$\geq$1, has a distinct roots.

UNIT-IV

TWO MARK QUESTIONS

1. Define commentator group.
2. Define solvable group.
3. Define n^{th} root of a polynomial.
4. Define solvable.

FIVE MARK QUESTIONS

1. If F has all its n^{th} root of unity and a$\neq$0 in F, x^n-a$\in$F[x] and K be the splitting field F, then prove that

i) K= F(u) where uis any root x^n-a.

ii) The Galois gorsy of x^n-a over F is abelian.

2. Prove that the polynomials of degree ≥ 5 is not solvable by radicals.

TEN MARK QUESTIONS

1. If p(x) $\in$ F[x] is solvable by radicals over F, then prove that the Galois group over F of p(x) is a solvable group.
2. State and prove wedder burn theorem.
3. Prove that the general polynomial of degree n ≥ 5 is not solvable by radicals
4. Let D be a division ring such that to every a $\in$ D there exist a positive integer n(a)>1, depending on a, such that a $^{n(a)}$=a, prove that D is commutative field.
5. If F has all its n^{th} root of unity and a $\neq 0$ in F, x^n-a $\in$ F[x] and K be the splitting field F, then prove that

i) K= F(u) where uis any root x^n-a.

ii) The Galois group of x^n-a over F is abelian.

6. The general polynomials of deg n ≥ 5 is not solvable by radicals.
7. A finite divsions ring is necessarily a commutative field.

UNIT-V

TWO MARK QUESTIONS

1. Define algebraic over a field.
2. H is a subring of Q. If X ϵH, then prove that x^*ϵH and N(x) is a positive integer for every non zero x in H.
3. Define adjoint.

FIVE MARK QUESTIONS

1. For all x, y $\in$ Q , prove that N(x y)=N(x)N(y).
2. Let a and b be in H with b $\neq$ 0, prove that there exist two elements C and d in H such that a=cb+ d and N(a)<N(b).
3. Let C be the field of complex numbers and suppose the division ring D is algebraic over C . then prove that D=C.

TEN MARK QUESTIONS

1. State and prove the left –division algorithm lemma.
2. i) prove that the adjoint in Q satisfies

   i) $x^{**}=x$

   ii) $(\delta x+vy)^{*}=\delta x^{*}+vy^{*}$

   iii) $(x\,y)^{*}=y^{*}x^{*}$

   ii) If C is the field of complex numbers and the division ring D is algebraic over C, then prove that D=C.

3. prove that N(xy)=N(x)N(y) for all x,y∈Q
4. prove that the adjoint in Q satisfies

   i) $x^{**}=x$

   ii) $(\delta x+vy)^{*}=\delta x^{*}+vy^{*}$

   iii) $(x\,y)^{*}=y^{*}x^{*}$ for all x,y is Q and all real δ and γ

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