Department of Mathematics

QUESTION BANK

Class: I M.Sc Mathematics
Sub Name: NUMERICAL ANALYSIS
Sub Code: [PMT808S]
Staff Name: Mr.T.Henson

UNIT-1

2Marks:

1. Write the Formula for Fixed Point Iteration Method.
2. Write the Formula for Newton's Raphson Method.
3. Explain Secant Method.
4. Write the Formula for Regula-Falsi Method.

5Marks:

1. Explain Newton's Raphson Method.
2. Find a real root of $3x - \cos x - 1 = 0$, using fixed point method correct to 5 decimal places.
3. Find a real root of $x^3 - 2x - 8 = 0$, using Newton's Raphson Method correct to 5 decimal places.
4. Find a real root of $x^3 - 4x - 5$, by N-R scheme to 5 decimal Places.
5. Find a real root of $x^3 + x^2 - 1 = 0$ correct to 5 decimal places.

6. Find a real root of $3x - \cos x - 1 = 0$ by Newton's Raphson Method correct to 4 decimal places.

7. Explain Bisection Method.

8. Compute the positive roots of $x^3 - 2x - 8 = 0$ by using regula-falsi method.

9. Compute the real roots of $x^3 + x^2 - 1 = 0$, by using regula-falsi method.

10.Find the positive roots of $3x - \cos x - 1 = 0$, by using Regula-Falsi method.

11. Find the real root of $x^3 - 2x^2 - 3x - 10 = 0$,by using regula-falsi method.

12. Explain the convergence of fixed point iteration method.

13.Derive the convergence when roots are repeated.

10MARKS:

1.Find the Real root of $x^3 - 2x - 8 = 0,$ using fixed point method, using fixed point method correct to 5 decimal places.

2. Find a real root of $x^3 - 4x - 5 = 0,$ by fixed point iteration method, correct to 5 decimal places.

3. Find a Real root of $x^3 + x^2 - 1 = 0$ by fixed point point iteration method correct to 5 decimal places.

4. Find the positive root of the equation $x^2 - 6e^{-x} = 0,$ by Regula-falsi method correct to 3 decimal places.

5. Find a real root of $x^3 - 5x + 3 = 0$, using secant method correct to 3 decimal places lies between (1 & 2).

6. Find the real root of $x^3 - 5x + 6 = 0$, using secant method correct to 5 decimal places.

7. Find a real root of the equation $x^3 - 2x - 5 = 0$, using bisection method correct to 4 decimal places.

8. Find the real root of $x^3 - 2x - 8 = 0$, using bisection method to correct to 5 decimal places.

9. Using the bisection method, find the positive root of $x^3 - 2x^2 - 3x - 10 = 0$ correct to 3 decimal places.

10. Establish the condition for convergence of fixed point method.

11. Explain the convergence of secant/ Regula falsi method.

12. Establish convergence of Newton's Raphson method.

UNIT-2

2 Marks:

1. Define Simpson's (1/3) rule.

2. Define Trapezoidal rule.

3. Define Simpson's (3/8) rule.

4. Evaluate I= $\int_0^{1.2} e^{-x^2} dx$ using simpon's rule with h=0.2.

5. Write the formula for open type formula.

5MARKS:

1. Evaluate $\int_0^1 e^{-x^2} dx$ by dividing the range of the integration into 4 equal parts using
   a) Trapezoidal rule

b) Simpson′s $(1/3)$ rule

c) Simpson′s $(3/8)$ rule

2. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using trapezoidal rule with h=0.2.Hence determine the value of π.

3. Evaluate $\int_{0.2}^{1.4} (\sin x - l_n x + e^x)$ by simpson's (1/3) rule by taking h=0.1.

4. Evaluate $\int_0^2 \frac{dx}{x^3+x+1}$ to 3 decimal places the range of integration into 8 equal parts using Simpson's rule.

5. Evaluate the integral $I = \int_0^1 \frac{1}{1+x} dx$ taking h=0.5, Trapezoidal rule.

6. Evaluate the integral $I = \int_0^1 l_n \; x \, dx$ dividing the integral into four equal parts compare with exact solution.

7. Show that the sum of the cubes of first 'n' natural numbers $1^3 + 2^3 + 3^3 + \dots\dots\dots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$

8. Find the sum of the series using Euler Maclaurin's formula upto thirdderivative only $\frac{1}{1.00} + \frac{1}{1.01} + \frac{1}{1.02} + \frac{1}{1.03} + \dots\dots\dots\dots + \frac{1}{2.00}$

9. Evaluate the integral $I = \int_0^1 \frac{dx}{\sqrt{1-x}}$

10MARKS:

1. Derive the Newton-cotes Formula.

2. Derive the Euler Maclaurian′s Formula.

3. Using Euler Maclaurian′s formula, compute the value of the integral $I = \int_0^{1.0} \left[\sin x - l_n(1+x) + e^x\right] dx$ dividing into five equal parts.

4. Determine the value of integral $I = \int_0^1 \left[x\, l_n\,(1+x)\, dx\right]$ using Euler Maclaurian′s formula after dividing the integral into 5 equal parts. Compute upto 5 decimal places and use formula upto first derivatives only.

5. Compute $I = \int_0^1 e^{2x}\, dx$ by Romberg integration method correct upto 3 decimal places using trapezoidal rule.

6. Compute $I = \int_0^1 e^x\, dx$ by Romberg integration by using trapezoidal rule correct to 3 decimal places. Compare with exact solution.

7. Find the value of the integral $I = \int_0^1 \sqrt{1+2x}\, dx$ by Gaussian Quadrature using

   a) 2- point formula
   b) 3- Pointformula; Compare the results with exact solutions.

8. Evaluate the Integral $I = \int_0^2 \sqrt{1+4x}\;\, dx$

   a) Gaussian 2- point formula
   b) Gaussian 3- point formula
   c) Simpson′s rule with two intervals
   d) Simpson′s rule with four intervals. Compare the results with exact value.

9. Find the value of the integral $I = \int_1^2 e^{2x}\, dx$ by Gaussian quadrature using
   a) 2- point
   b) 3- Point formula. Compare the result with exact solution.

UNIT-3

2MARKS:

1. Define Spline .
2. Define Piece-wise Polynomial.
3. Write the governing equations of Construction of First derivative form of cubic Spline.
4. Write the governing equations of Construction of Second derivative form of cubic Spline.

5MARKS:

1. Find the cube-root of 21 by fitting a natural cubic spline to the following table of values of cube root.

x	0	1	8	27
$f(x)=x^{1/3}$	0	1	2	3

2. Explain the Uniqueness of cubic spline.

10MARKS:

1. Derive the construction of cubic spline (second derivative form).

2. Derive the construction of cubic spline (first derivative form).

3. Using Cubic spline find the value of function $f(x) = l_n\ x$ for $x = 1.8$ from the following data:

x	1.0	1.2	1.6	2.0
$f(x)$	0.0	0.18232	0.47000	0.69315

4. Using a cubic spline, interpolate the value of the periodic function $f(x) = \cos x$ at $x = \frac{2\pi}{3}$ when the function values are tabulated as given below:

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$f(x)$	1	0	-1	0	1

5. Using Natural cubic spline find the value of function for f(x) at x=0.8 from the following data:

x	0	0.2	0.6	1.0
$f(x)$	0	0.1823	0.47000	0.6932

UNIT-4

2MARKS:

1. State the minimal property of cubic spline.

2. Write the formula for solving Differential equation using spline?

3. Express x^5 by chebyshev series without using recurrence relation. Use appropriate formula.

5MARKS:

1. Explain the minimum property of cubic spline.

2. Explain the Parametric form of cubic spline.

3. Express x^4 and x^5 by chebyshev without using recurrence relation. Use appropriate formula.

4. Express the function $f(x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}$ by a chebyshev series in the interval $-1 \le x \le 1$. Use appropriate formula and not the recurrence relation.

5. Express $\cos^{-1} x$ by chebyshev series. Hence prove that

$$\frac{\pi^2}{8}=1+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+..........$$

10Marks:

1. Given the differential equation.

$\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+4x^2y=0,\ y(0)=3$ And $y(0.4)=2.46884,$

Find $y(0.2)$ using a cubic spline by dividing the interval (0, 0.4) into two equal subbintervals.

2. The coordinates of four points on a curve are given as A (1, 0), B (0, 1), C (-1, 0) and D (0, -1). The closed curve ABCDA is to be a approximated by fitting a cubic spline
   Taking θ as a parameter where θ denotes an angle with the positive x-axis. Obtain $\frac{dx}{d\theta}$ and $\frac{dy}{d\theta}$ at these points and compute the values of x and y for $\theta=\frac{2\pi}{3}$.

3. Four points A(1, 0), B(0, 1), C(-1, 0) and D(0, -1) lie on a closed curve ABCDA in $x-y$ plane. Find the cubic splines for x and y between the points B and C taking the parameter s where s denotes the cummulative length on the curve meaured in the anticlockwise direction starting with $s=0$ on A. Find x and y for $s=\frac{4\sqrt{2}}{3}$.

4. Four points A(1, 0), B(0, 1), C(-1, 0) and D(0, -1) are given on a closed curve ABCDA in the $x-y$ plane. Using parmeter $u, 0 \le u \le 1$ in each interval find the cubic spline $\mathrm{P}(u)$ between the points B and C. Hence find the values of x and y for $u=\frac{1}{3}$.

5. Express the function $f(x)=\cos^{-1} x$ by truncated chebyshev series $\cos^{-1} x = a_0 + a_1 T_1(x) + a_3 T_3(x)$. Compute a_0, a_1, a_3 numerically taking nodal points as zeros of $T_5(x)$.

6. Derive the Minimal property of Cubic spline.

UNIT-5

2MARKS:

1. Classify the PDF a $\frac{\partial^2 u}{\partial x^2} + b\frac{\partial^2 u}{\partial x \partial y} + C\frac{\partial^2 u}{\partial y^2} + e = 0.$

2. Define parabolic equation.

3. Define Elliptic equation.

4. Define Hyperbolic equation.

5. List the comparison of explicit form, fully implicit form and implicit form.

5MARKS:

1. Explain Crank-Nicolson′s scheme.

2. Given the heat conduction equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $0 \le x \le 0.5, t > 0$ with boundary conditions $u(0,t) = 0$ and $\frac{\partial u}{\partial x} = 4$ at $x = 0.5$ and initial condition $u(x,0) = 4x^2$ .Solve the problem dividing the interval [0,0.5] into two equal parts, by C-N method fot t=0 (0.) 0.2. Approximate the derivative boundary condition by backward difference.

3. Write about two methods of solving parabolic equation.

10MARKS:

1. Find the numerical solutions of the heatconduction equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}; 0 \le x \le 1, t > 0$ with boundary conditions $u(0,t) = u(1,t) = 1$ and the initial condition

$$u(x,0) = \begin{cases} 1 + 2x \ , & 0 \le x \le \frac{1}{2} \\ 3 - 2x \ , & \frac{1}{2} \le x \le 1. \end{cases}$$

Use explicit method taking $\Delta x = 0.2, \Delta t = 0.02$ and compute upto $t = 0.24$ up to six decimal places.

2. Show by solving the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}; 0 \le x \le 1, t > 0$ with boundary conditions $u(0,t) = u(1,t) = 1$ and the initial condition

$$u(x,0) = \begin{cases} 1 + 2x \ , & 0 \le x \le \frac{1}{2} \\ 3 - 2x \ , & \frac{1}{2} \le x \le 1. \end{cases}$$

Taking $\Delta x = 0.2, \Delta t = 0.04$ (and computing upto $t = 0.24$) upto six decimal places that the explicit scheme is unstable.

3. Find the numerical solution of the heat conduction equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$; $0 \le x \le 1, t > 0$ with boundary conditions $u(0,t) = u(1,t) = 1$ and the initial condition

$$u(x,0) = \begin{cases} 1 + 2x \ , & 0 \le x \le \frac{1}{2} \\ 3 - 2x \ , & \frac{1}{2} \le x \le 1. \end{cases}$$

Use C-N scheme and compute upto $t = 0.24$.

4. Solve the problem given that the heat conduction equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ $(0 \le x \le 0.5), t > 0$ with boundary condition $u(0,t) = 0$ and $\frac{\partial u}{\partial x} = 4$ at $x = 0.5$ and initial conditions $u(x,0) = 4x^2$ replacing the boundary condition at $x = 0.5$ by central difference.

5. Solve the problem given that the heat conduction equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ $(0 \le x \le 0.5), t > 0$ with boundary condition $u(0,t) = 0$ and $\frac{\partial u}{\partial x} = 4$ at $x = 0.5$ and initial conditions $u(x,0) = 4x^2$ by C-N method with $\Delta t = 0.1$ by backward difference.

6. Solve the problem given that the heat conduction equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ $(0 \le x \le 0.5), t > 0$ with boundary condition $u(0,t) = 0$ and $\frac{\partial u}{\partial x} = 4$ at $x = 0.5$ and initial conditions $u(x,0) = 4x^2$ with $\Delta t = 0.1$ and replacing the derivative at the boundary by central difference.