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SUB CODE: PMT809T
Subject name:FLUID DYNAMICS
Department; Mathematics
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UNIT-I

2MARKS:

1. Define Real fluid and ideal fluid.
2. Define stream lines.
3. Define path line.
4. Define steady flow.
5. Define unsteady flow.
6. Define stream tube
7. Define potential kind.
8. To show that the steady flow $\psi =$ constant represents the stream line.
9. Define Velocity vector.
10. Define vortex tube & vortex filament.
11. Define circulation.
12. Define the Equation of continuity.
13. Show that the $\vec{q} = (-wy, wx, 0)$ $w =$ constant is not of the potential kind.
14. Define velocity potential.
15. Define Axi-symmetric flow.
16. What is the image of a doublet in a rigid infinite plane?

5MARKS:

1. Derive the velocity of a fluid at apoint.

2. Derive the velocity potential. To show that in a steady flow ϕ represent equi-potential curve.

3. Show that the stream lines in the curves of the equi-potential are orthogonal to one another.

4. Derive the local & particle rate of changes.

5. For an incompressible fluid $\vec{q} = [-wy, wx, 0]$, $w =$ constant. Discuss the nature of the flow.

6. Find the path of the fluid particle is $u = \frac{x}{1+t}, \quad v = \frac{y}{1+t}, \quad w = \frac{z}{1+t}$.

7. Find the stream line for $\vec{q} = -3y^2\vec{i} - 6x\vec{j}$ at (1, 1).

8. Determine the acceleration at a point (2, 1, 3) at $t = 0.5$ secs, $u = yz + t, \quad v = 2z - t, \quad w = xy$.

9. Determine the acceleration of fluid particle from the flow fluid $\vec{q} = Ax^2y\vec{i} + By^2zt\vec{j} + zt^2\vec{k}$.

10MARKS:

1. At a point in an incompressible fluid having spherical polar coordinates (r, θ, ψ), the velocity components are $2Mr^{-3}\cos\theta$, $Mr^{-3}\sin\theta$, 0, where M is a constant.
   i) Show that the velocity of the potential kind.
   ii) Find the velocity potential and the equation of stream line.

2. Derive the equation of continuity.

3. Test whether the motion specified by $q = k^2\frac{(x\bar{j} - y\vec{i})}{x^2 + y^2}$, $k =$ constant, is a possible motion for an incompressible fluid if so determine the equation of the stream line

also test whether the motion is the potential kind and so determine the velocity potential.

4. For a fluid moving in a fine tube of variable section A. Prove the first principle that the equation of continuity is $A\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial s}(A\rho v)=0$, where v is the speed at the point P of the fluid and S be the length of the tube uptoP.What does this become for steady incompressible fluid?

5. Liquid flows through a pipe whose surface is the surface of revolution to the curve $y=a+\frac{kx^2}{a}$ about the $x-axis$ $(-a \le x \le a)$. If the liquid enters at the end $x=-a$ of the pipe with velocity v, show that the time taken by a liquid to transverse the entire length of the pipe from $x=-a$ to $x=a$ is $\frac{2a}{v(1+k^2)}\left(1+\frac{2}{3}k+\frac{1}{5}k^2\right)$.

6. Derive the Acceleration of the fluid.

7. Derive the conditions at a rigid boundary.

8. Show that a fluid of constant density can have a velocity $\vec{q}=\left[\frac{-2xyz}{(x^2+y^2)^2}, \frac{(x^2-y^2)z}{(x^2+y^2)^2}, \frac{y}{x^2+y^2}\right]$ find the velocity vector. Also show that the motion is irrational.

9. Determine the acceleration of the fluid particle from the flow fluid $\vec{q}=(Ax^2yt)\vec{i}+(Bx^2yt)\vec{j}+(cxyz)\vec{k}$.

UNIT-II

2MARKS:

1. Define hydrostatic pressure.

2. What are the important rules in hydrostatic.

3. Define conservative force.

4. Define Beltrami vector.

5. Define Bernoulli′s equation.

5MARKS:

1. Prove that the pressure P is small in all directions at a boundary of two inviscoidimmisible fluid.

2. Explain the construction of pitete tube.

3. Explain the construction of venturi fluid.

4. Derive the Bernoulli′s equation.

10MARKS:

1. Derive the pressure at apoints in a moving fluid.

2. Derive the Euler′s equation of motion.

3. AB is a tube of small uniform force forming a quadrature arc of a circle of radius 'a' and center 'o'. OA being horizontal and OB vertical with B below O. The tube is full of density ρ. The end B being closed. If B is suddenly open show that initially $\frac{du}{dt}=\frac{2g}{\pi}$ where $\mu = u(t)$ is the velocity and that the pressure at apoint whose angular distance from A is θ immediately drops to $\rho\, g\, a\left(\sin\theta - \frac{2\theta}{\pi}\right)$ above atmospheric pressure prove further that when the liquid remaining in the tube subtends an angle β at the center, $\frac{d^2\beta}{dt^2} = -\frac{2g}{a\beta}\sin^2\left(\frac{\beta}{2}\right)$.

4. A long is of length l and has slowly-tapering cross section. It is inclined at an angle α, to the horizontal and water flows steadily through it from the upper to the lower end the section at the upper to the end has twice the radius of the lower end. At the lower end, the wateris delivered at atmospheric pressure. If the pressure the upper and it's twice at atmospheric. Find the velocity of delivery.

5. Discussion of the case of steady motion under conservative body force.

6. A velocity potential for the motion due to the sphere of radius 'a' moving with variable v in the direction $\theta = 0$ in an infinite uniform inviscoid incompressible fluid of density ρ is $\phi = \frac{-ua^3\cos\theta}{2r^2}$. Assuming suitable form of Bernoulli's equation deduce that the pressure at point on the sphere is given by

$\frac{P}{\rho} = \frac{P_0}{\rho} + g\,z - \frac{1}{8}v^2\left(5 - 9\cos^2\theta\right) + \frac{1}{2}ua\cos\theta$ where the only body force is gravity and z is the vertical co-ordinates measured downwards from a fixed origin.

UNIT-III

2MARKS:

1. Define simple sources.
2. Define simple sinks.
3. Define doublets.
4. Write the vector moment of the doublet.
5. Define Axis-symmetrical flows.

5MARKS:

1. Find the velocity potential due to the doublet at 'o'.
2. Discuss doublet in a uniform stream.

10MARKS:

1. Find the velocity potential ϕ for a source of strength 'm' at the origin.

2. Doublets of strength μ_1, μ_2 are situated at points A_1, A_2 whose Cartesian co-ordinates are $(0, 0, c_1)$ & $(0, 0, c_2)$ their across being directed toward end away from the origin respectively. Find the conditions that there is no transport of fluid cover the surface of the sphere $x^2 + y^2 + z^2 = c_1 c_2$.

3. Prove that the velocity potential at a point P due to an uniform finite lines source AB of strength 'm' per unit length is of the form $\phi = m \log f$, where $f = \frac{r_2 - x_2}{r_1 - x_1} = \frac{r_1 + x_1}{r_2 + x_2} = \frac{a+l}{a-l}$ in which $AB = 21, PA = r_1, PB = r_2, NA = x_1, NB = x_2, N$ being the foot of the perpendicular from 'P' on the line AB and $2a$ the length of the major axis of the spheroid through 'P' having A, B as foci. Show that the velocity at P is $\frac{(2ml\cos\alpha)}{(a^2 - l^2)}\overset{\rho}{u}$, where u is the unit vector along the normal tp the spheroid through at P and $2\alpha = APB$.

4. Discuss the image of a rigid infinite plane.

5. A three dimensional doublet of strength μ whose axis is in the direction $\overrightarrow{OX}$ is distant 'a' from the rigid plane $x = 0$ which is the whole boundary of liquid of density ρ, infinite in extent. Find the pressure at apoint on the boundary distant 'r' from the doublet given that the pressure at α is P_α. Show that the pressure on the plane is least at a distance $\frac{a\sqrt{5}}{2}$ from the doublet.

6. Find the expression for the velocity components in terms of ψ.

7. Find the velocity components in the polar co-ordinates (r, θ).

8. Derive the Special forms of the stream functions for axis-symmetric irrational motion.

9. Define the stokes stream function $\psi(r, \theta)$ in spherical polar co-ordinates (r, θ, ψ) for the axis symmetric flow an incompressible fluid. Determine the stream function corresponding
   i) To a uniform stream parallel to axis $\theta = 0$.
   ii) To a spherically symmetric radical velocity fluid from a point source at a origin outward flux being $4\pi m$,

The equation $r \sin\theta = 2a \cos\theta/2$ represents the surface of a rigid blunt closed cylinder symmetric about the axis $\theta = 0$, inviscoid the flow irrationally part this cylinder the velocity for from the cylinder being 'u' parallel to the cylinder axis. Show that the sum of the two stream function i) & ii) above with $m = a^2 u$ may be used to represent this flow. Find the pressure distribution as a function of θ on the cylinder surface.

UNIT-IV

2MARKS:

1. Define two dimensional flows.

2. Define stream function.

3. Define line source.

4. Define line sink.

5. What is the image of a line source in a rigid infinite plane?

6. State Milne-Thomson circle theorem.

7. Define Magnus effect.

5MARKS:

1. Derive the uniform flow past a fixed infinite circular cylinder.

2. Discuss the flow for which $w = z^2$.

3. Explain the line doublets.

4. Discuss the flow due to a uniform line doublet at O of strength μ per unit length, its axis being along $\overrightarrow{OX}$.

5. Discuss the Two dimensional image systems.

6. Discuss the problem of uniform flow part a stationary cylinder and uniform stream at incidence at OX.

7. Write down the complex potential for the flow due to the uniform stream which is incident to the positive axis at angle α.

8. Obtain the complex velocity potential for line doublet parallel to the axis a rigid circular cylinder.

9. Find the equation of the stream lines due to a uniform line source of strength m through the points A(-c,0) & B(c,0) and a uniform line sink of strength $2m$ through the origin.

10MARKS:

1. Write the uses of cylindrical polar co-ordinates.

2. A cylinder of infinite length nearly circular section moves through an infinite volume of liquid with a velocity U at right angles to its axis and in the direction of $x-$axis. If its section is specified by the equation $R = a(1 + \varepsilon \cos n\theta)$, where ε is small, show that the approximate value of the velocity potential is
$$Ua\left\{\frac{a}{R}\cos\theta + \varepsilon\left(\frac{a}{R}\right)^{n+1}\cos(n+1)\theta - \varepsilon\left(\frac{a}{R}\right)^{n-1}\cos(n-1)\theta\right\}.$$

3. Discuss the stream function for the two dimensional steady motion.

4. Derive the complex potential for two dimensional irrational incompressible flows.

5. Derive the complex velocity potential for standard two dimensional flows.

6. Explain the strength of the line source.

7. State and prove Milne-Thomson circle theorem.

8. Describe the irrational motion of an incompressible liquid for which the complex potential is $w = i\,k \log z$. Two parallel vertices of strength k_1, k_2 $(k_1 + k_2 \neq 0)$ in unlimited liquid cross the $Z-$planes at points A, B respectively. The center of mass of masses k_1 at A and k_2 at B in G

i) Show that the motion of the liquid is due slowly to these vertices, G is a fixed point about which A, B moves in a circle with angular velocity $\dfrac{(k_1 + k_2)}{(AB)^2}$.

ii) Show that the fluid speed at any point P in the $Z-$plane is $(k_1 + k_2)\dfrac{CP}{AP.BP}$.

9. A Two dimensional doublet of strength $\mu\hat{r}$ is at the point $z = ia$ in a stream of velocity $-v\hat{i}$ in a semi-infinite liquid of constant density occupying the half plane $y > 0$ showing $y = 0$ as a rigid boundary [$\hat{i}$ I a unit vector in the positive $x-$axis]. Show that the complex potential of the motion is $w = \dfrac{v_z + 2\mu z}{(z^2 + a^2)}$. Show that for $0 < \mu < 4a^2 v$, there are no stagnation points on this boundary and that the pressure on it's a minimum at the origin and a maximum at the points $Z = \pm a\sqrt{3}$.

10. Drive the image of a line source in a circular cylinder.

11. A vertex of circulation $2\pi k$ is at rest of a point $Z = na$ $(n > 1)$ in the presence of a plane circular boundary $|z| = a$; around which there is a circulation $2\pi\lambda k$. Show that $\lambda = \dfrac{1}{(n^2 - 1)}$. Show that there are two stagnation points on a circular boundary $Z = ae^{i\theta}$, symmetrically placed about the real axis in the quadrants nearest to the vortex given by $\cos\theta = \dfrac{3n^2 - 1}{2n^3}$ and prove that θ is real.

12. Discuss the problem for infinite circular cylinder uniform stream with circular.

13. A source and sink of equal strength are placed at points $\left(\pm\frac{1}{2}a, 0\right)$ within a fixed circular boundary $x^2 + y^2 = r^2$. Show that the strealines are given by $\left(r^2 - \frac{a^2}{4}\right)\left(r^2 - 4a^2\right) - 4a^2y^2 = ky\left(r^2 - a^2\right)$.

UNIT-V

2MARKS:

1. Define Stress Matrix.

2. What is stress tensor?

3. Define principal stress.

5MARKS:

1. Write down the stress component in a real fluid.
2. Explain the rate of strain quadratic and principal stress.
3. Derive the translational motion of fluid element.
4. Express the six distinct components of the stress matrix of the principal stresses.

10MARKS:

1. Discuss the relation between Cartesian and stress.
2. Write down the properties of the rate of strain quadric.
3. Derive the relation between stress and rate of strain.
4. Derive the co-efficient of viscosity and laminar flow.
5. Derive the Navies-stokes equation of a motion of a viscous fluid.