Reg. No									

St. Joseph's College of Arts & Science (Autonomous)
St. Joseph's College Road, Cuddalore – 607001

MT510 – REAL ANALYSIS – I (2018 Batch only)

Time : 3 hrs **Max Marks :75**

Please send your soft copy of Answer Booklet to this Mail ID

venkatesan@sjctnc.edu.in

SECTION – A (5X2=10)

Answer **ALL** Questions

1. Define countable set.
2. Prove that the sequence $\{\log(\frac{1}{n})\}_{n=1}$ diverges to minus ∞.
3. Define limit inferior of the sequence.
4. If $\lim_{x\to a} f(x)$ = L and $\lim_{x\to a} g(x)$ = M, then $\lim_{x\to a}[f(x) + g(x)]$ = L + M.
5. Define open subset of a metric space M.

SECTION – B (3X5=15)

Answer any **THREE** Questions

6. If A is any nonempty subset of R that is bounded below, then A has a greatest lower bound in R.
7. Prove that all subsequences of a convergent sequence of real numbers converge to the same limit.
8. If $\lim_{n\to\infty} s_n$ = L, then $\lim_{n\to\infty} s_n^2 = L^2$.
9. Let < M, ρ > be a metric space and let a be a point in M. Let f and g be real valued functions whose domains are subsets of M. If

$\lim_{x\to a} f(x)$ = L, and $\lim_{x\to a} g(x)$ = N, then prove that $\lim_{x\to a} f(x)g(x)$ = LN.

10. I f $\mathcal{F}$ is any family of closed subsets of a metric space M, then $\cap_{F\in\mathcal{F}}$ F is also closed.

SECTION - C (5X10=50)

Answer **ALL** Questions

11. a) If $A_1, A_2, \ldots\ldots\ldots$ are countable sets , then $\cup A_n$ is countable.

(or)

b) Prove that the set of all rational numbers is countable.

12. a) Prove that a non-increasing sequence which is bounded below is convergent.

(or)

b) Prove that the sequence $\{ (1+\frac{1}{n})^n \}_{n=1}^{\infty}$ is convergent.

13. a) If $\lim_{n\to\infty} s_n = L$,then prove that $\lim_{n\to\infty} s_n^2 = L^2$.

(or)

b) Prove that the series $\sum_{n=1}^{\infty}(\frac{1}{n})$ is divergent.

14. a) Let $\sum_{n=1}^{\infty} a_n$ be a series of nonzero real numbers and let $a = \lim_{n\to\infty} inf\ |(a_{n+1}/a_n)|$ and $A = \lim_{n\to\infty} sup\ |(a_{n+1}/a_n)|$, (so that $a \leq A$) . Then

i) If $A < 1$, then $\sum_{n=1}^{\infty} a_n$ converges absolutely;

ii) If $a > 1$, then $\sum_{n=1}^{\infty} a_n$ diverges;

iii) If $a \leq 1 \leq A$, then the test fails.

(or)

b) Let $< M, \rho >$ be a metric space and let a be a point in M. Let f and g be real valued functions whose domains are subsets of M. If $\lim_{x\to a} f(x)$ = L and $\lim_{x\to a} g(x)$ = N, then prove that i) $\lim_{x\to a} f(x)g(x)$ = LN. ii) $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{L}{N}$, if $N \neq 0$.

15. a) Let $< M_1, \rho_1 >$ and $< M_2, \rho_2 >$ be metric spaces and let $f:M_1 \to M_2$. Then f is continuous on M_1 if and only if $f^{-1}(G)$ is open in M_1 whenever G is open in M_2.

(or)

b) Let $< M_1, \rho_1 >$ and $< M_2, \rho_2 >$ be metric spaces and let $f: M_1 \rightarrow M_2$. Then f is continuous on M_1 if and only if $f^{-1}(F)$ is closed subset of M_1 whenever F is closed subset of M_2.

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