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REDUCED DIFFERENTIAL TRANSFORM METHOD FOR SOLVING PARTIAL DIFFERENTIAL EQUATIONS ARISING IN PHYSICS & ENGINEERING

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Abstract

In this paper, we apply Reduced differential transform method (RDTM) to obtain exact solutions for the General linear initial value problems and First order linear transport equations .The numerical results show that the method reduces the numerical calculations with approximate solutions. The results reveal that Reduced differential transform method (RDTM) is very effective, convenient, and quite accurate to systems of linear equations in various field of Science and Engineering

Keywords: Reduced differential transform method (RDTM), General linear initial value problems, Transport Equations.

INTRODUCTION

Partial differential equations (PDEs) have numerous essential applications in various fields of science and engineering such as fluid mechanic, thermodynamic, heat transfer, physics. Many physical problems can be described by mathematical models that involve partial differential equations. A mathematical model is a simplified description of physical reality expressed in mathematical terms. Most of these equations are very difficult to solve for their exact analytical solutions except in few cases. There are several numerical methods developed for solving partial differential equations with variable coefficients such us He's Polynomials, elementary method, homotopy analysis method and the modified variational iteration method, method of characteristics. Therefore, we often times attempt to develop new techniques to enable us obtain an approximate solution as close as possible to the exact ones.

The main goal of this paper is to apply the reduced differential transform method (RDTM) to obtain the exact solution for the General linear initial value problems,

$$a(x,t)\, u_x + b(x,t)\, u_t = c(x,t)u + d(x,t)$$

with initial condition $u(x,0)=f(x)$ and

First Order Linear Transport Equation

$u_t + cu_x = 0$,where u is the function of x and t ie. $u=u(x,t)$ and wave speed c is a constant.

with initial condition $u(x,0)=f(x)$

This method, like the differential transform method, was first introduced by Zhou in 1986 and the variational iteration method introduced by him has been used by many mathematicians and engineers to solve various functional equations. The main advantage of the method is the fact that it provides its user with an analytical approximation, in many cases an exact solution, in a

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rapidly convergent sequence with elegantly computed terms. Analytical solutions enable researchers to study the effect of different variables or parameters on the function under study easily. Its small size of computation in comparison with the computational size required in other numerical methods, and its rapid convergence show that the method is reliable and introduces a significant improvement in solving wave equations over existing methods. The solution procedure of the RDTM is simpler than that of traditional DTM, and the amount of computation required in RDTM is much less than that in traditional DTM. The solution obtained by the reduced differential transform method is an infinite power series for initial value problems, which can be, in turn, expressed in a closed form, the exact solution.

ANALYSIS OF METHOD

In this section, we will give the methodology of the RDTM. First let's consider a function of two variables $u(x,t)$ and suppose that it can be represented as a product of two single-variable functions, $u(x,t)= f(x).g(t)$.

Based on the properties of differential transform function u(x,t) can be represented as

$$u(x,t) = \sum_{i=0}^{\infty} F(i)\, x^i \sum_{j=0}^{\infty} G(j)\, t^j$$

where $U_k(x)$ is called t-dimensional spectrum function of $u(x,t)$.

The basic definitions and operations of reduced differential transform method are introduced as follows:

Definition: 1

If the function $u(x,t)$ is analytic and differentiated continuously with respect to time t and space x in the domain of interest, then let

$$U_k(X) = \frac{1}{k!}\left(\frac{\partial^k}{\partial t^k}u(x,t)\right)_{t=0} \qquad [1]$$

where the t-dimensional spectrum function $U_k(x)$ is the transformed function. In this paper, the lowercase $u(x,t)$ represent the original function, while the uppercase $U_k(x)$ stand for the transformed function.

Definition: 2

The differential inverse transform of $U_k(x)$ is defined as follows:

$$u(x,t) = \sum_{k=0}^{\infty} U_k(X)\, t^k \qquad [2]$$

Then combining equation (1) and (2) we can write

$$U(x,t) = \frac{1}{k!}\sum_{k=0}^{\infty}\left(\frac{\partial^k}{\partial t^k}u(x,t)\right)_{t=0} t^k \qquad [3]$$

From the above definitions, it can be found that the concept of the reduced differential transform is derived from the power series expansion. To illustrate the basic concepts of RDTM, consider the following general linear initial value problem

$$a(x,t)Ru(x,t)+b(x,t)Lu(x,t)=c(x,t)u(x,t)+d(x,t)$$

with initial condition $u(x,0)=f(x)$ and

First order linear transport equation

$$Lu(x,t)+c\, Ru(x,t) =0$$

With initial condition $u(x,0) = f(x)$,where $L= \partial/\partial t$, $R=\partial/\partial x$ is a linear operator .

According to the RDTM and Table 1(given below), we can construct the following iteration formulas:

$$b(x,t)(k+1)U_{(k+1)}(x)=-a(x,t)(\partial/\partial x)U_k(x)+c(x,t)U_k(x)+d(x,t) \qquad [4]$$

With initial condition $U_0(x) =f(x)$ and [5]

$$(k+1)U_{(k+1)}(x)+c(\partial/\partial x)U_k(x)=0 \qquad [6]$$

With initial condition $U_0(x) =f(x)$ **[7]**

Where $U_k(x)$, $RU_k(x)$ are the transformations of the functions Lu(x,t), Ru(x,t) respectively.

Substituting (5) into (4) and (7) into (6) by straightforward iterative calculations, we get the following $U_k(x)$ values. Then the inverse transformation of the set of values $\{U_k(x)\}$ (k=0 to n) gives the approximation solution as,

$$u_n(x,t) \sum_{k=0}^{n} U_k(X)\, t^k$$

where n is order of approximation solution. Therefore, the exact solution of the problem is given by

$$u(x,t)=\lim_{n\to\infty} u_n(x,t)$$

We give a table which included fundamental transformation properties of RDTM in the following manner. Table 1: Basic transformations of RDTM for some functions

Functional Form	Transformed Form
$u(x,t)$	$U_k(x) = \frac{1}{k!}\left(\frac{\partial^k}{\partial t^k} u(x,t)\right)_{t=0}$
$w(x,t) = x^m\ t^n$	$w_k(x) = x^m \Delta(x-n)$, $\Delta(x-n) = \begin{cases} 1, k = n \\ 0, k \neq n \end{cases}$
$w(x,t)=x^m t^n u(x,t)$	$W_k(x)=x^m U_{(k-n)}(x)$
$w(x,t)=u(x,t)v(x,t)$	$W_k(x)=\sum U_i(x)V_{(k-i)}(x),(i=0 \text{ to } k)$
$w(x,t)=\partial^n/\partial t^n (u(x,t))$	$W_k(x)=(k+1)(k+2)\ldots\ldots(k+n)U_{(k+n)}(x)$
$w(x,t)=\partial^n/\partial x^n (u(x,t))$	$W_k(x,t)=\partial^n/\partial x^n\ U_k(x)$
$w(x,t)=(\partial^{n+m}/\partial x^n \partial t^m)(u(x,t))$	$W_k(x,t)=\partial^n/\partial x^n\{(k+n)!/k!\}U_k(x)$
$w(x,t)=\mu u(x,t)$	$W_k(x,t)=\mu U_k(x)$, μ is a constant

NUMERICAL APPLICATIONS

In order to assess the advantages and the accuracy of RDTM for solving General linear initial value problem and First order linear transport equations.

Example:3.1 consider the linear initial value problem and find exact solution

$$2x u_t + u_x = 2x u \qquad [8]$$

With initial condition $u(x,0)=x^2$ [9]

where $u=u(x,t)$ is a function of the variables x and t.Applying the basic properties of reduced differential transform to (8), we obtain the recurrence equation

$$2x(k+1)U_{(k+1)}(x) = -\partial/\partial x\ U_k(x) + 2xU_k(x) \qquad [10]$$

where the one-dimensional spectrum function is the transform function. From the initial condition (9), we write

$$U_0(x) = x^2 \qquad [11]$$

Substituting (11) into (10), we obtain the following $U_k(x)$ values successively

$U_1(x) = x^2-1$, $U_2(x) =(x^2/2)-1$,

$U_3(x) =(x^2/6 – ½),U_4(x) = (x^2/24)-1/6$ …and so on

Finally the differential inverse transform (2) of $U_k(x)$ gives

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x)t^k = x^2+(x^2-1)t+[(x^2/2)-1]t^2+[(x^2-6)-1/2]$$
$$t^3+[(x^2/24)-1/6]t^4....$$
$$=x^2\ \{1+t+t^2/2!+t^3/3!+t^4/4!.....\}$$
$$-t\ \{1+t+t^2/2!+t^3/3!+t^4/4!+.....\}$$
$$= x^2e^t–t\ e^t$$
$$u(x,t) = (x^2-t)\ e^t$$

which is the exact solution

Example :3.2 consider the linear initial value problem and calculate approximate or exact one

$$u_x + u=u_t \qquad [12]$$

with initial condition $u(x,0)=4e^{-3x}$ [13]

where $u=u(x,t)$ is a function of the variables x and t.

Applying the basic properties of reduced differential transform to (12), we obtain the recurrence equation

$$(k+1)U_{(k+1)}(x) = \partial/\partial x\ U_k(x) + U_k(x) \qquad [14]$$

where the one -dimensional spectrum function is the transform function. From the initial condition (13), we write

$$U_0(x) = 4e^{-3x} \qquad [15]$$

Substituting (15) into (14), we obtain the following $U_k(x)$ values successively

$U_1(x) =-8e^{-3x}$, $U_2(x) =8e^{-3x}$,

$U_3(x) =-(16/3)e^{-3x}$, $U_4(x) =(8/3)e^{-3x}$,

$U_5(x) =-16/15e^{-3x}$, $U_6(x) =(16/45)e^{-3x}$… and so on

Finally the differential inverse transform (2) of $U_k(x)$ gives

$$u(x,t) =\sum_{k=0}^{\infty} U_k(x)t^k$$
$$u(x,t) = 4e^{-3x}-8e^{-3x}t+8e^{-3x}t^2-16/3e^{-3x}t^3+ 8/3e^{-3x}t^4-16/15e^{-3x}t^5+.....$$
$$= 4e^{-3x}\{1-2t+2t^2-4t^3/3+2t^4/3-4t^5/15+4/45t^6-.....\}$$
$$= 4e^{-3x}\{1-2t+(2t)^2/2!-(2t)^3/3!+(2t)^4/4!-(2t)^5/5!+(2t)^6/6!-.....\}$$
$$= 4e^{-3x}e^{-2t}=4e^{-(3x+2t)}$$

Example: 3.3 consider the Transport Equation and calculate exact solution

$$u_t + u_x=0 \qquad [16]$$

With initial condition $u(x,0)=1/x^2$ [17]

where u=u(x,t) is a function of the variables x and t. Applying the basic properties of reduced differential transform to (16), we obtain the recurrence equation

$(k+1)U_{(k+1)}(x) = -\partial/\partial x\, U_k(x)$ **[18]**

where the one -dimensional spectrum function is the transform function. From the initial condition (17), we write

$U_0(x) = 1/x^2$ **[19]**

Substituting (19) into (18), we obtain the following $U_k(x)$ values successively

$U_1(x) = 2/x^3,\ U_2(x) = 3/x^4, U_3(x) = 4/x^5\ldots$ and so on

Finally the differential inverse transform (2) of $U_k(x)$ gives u(x,t)

$$u(x,t) = \sum_{k=0}^{\infty} U^k(x)t^k = 1/x^2 + (2/x^3)t + (3/x^4)t^2 + \ldots$$
$$= 1/x^2\{1+2(t/x)+3(t/x)^2+4(t/x)^3+\ldots\}$$
$$= 1/x^2(1-(t/x)\}^{-2}$$
$$= 1/x^2\{(x-t)/x\}^{-2}$$
$$= 1/(x-t)^2$$

which is the exact solution.

Example:3.4 consider the Transport Equation and find approximate or exact solution

$2u_t - 3/2\, u_x = 0$ **[20]**

with initial condition $u(x,o) = \cos 2x$ [21]

where u=u(x,t) is a function of the variables x and t. Applying the basic properties of reduced differential transform to (20), we obtain the recurrence equation

$2(k+1)U_{(k+1)}(x) = 3/2\ \partial/\partial x\, U_k(x)$ **[22]**

where the one -dimensional spectrum function is the transform function. From the initial condition (21), we write

$U_0(x) = \cos 2x$ **[23]**

Substituting (23) into (22), we obtain the following $U_k(x)$ values successively

$U_1(x) = (-3/2)\sin 2x$, $U_2(x) = (-9/8)\cos 2x$,

$U_3(x) = (9/16)\sin 2x$, $U_4(x) = (27/128)\cos 2x$

$U_5(x) = (-81/1280)\sin 2x \ldots$ and so on

Finally the differential inverse transform (2) of $U_k(x)$ give

$$u(x,t) = \sum U_k(x)t^k,\ (k=0 \text{ to } \infty)$$

$$u(x,t) = \cos 2x - (3/2)\sin 2x\, t - (9/8)\cos 2x\, t^2 + (9/16)\sin 2x\, t^3 + (27/128)\cos 2x\, t^4 - (81/1280)\sin 2x\, t^5 - \ldots$$

$$= \cos 2x\,\{1-(9/8)t^2+(27/128)t^4 - \ldots\} - \sin 2x\{(3/2)t-(9/16)t3+(81/1280)t5-\ldots\}$$
$$= \cos 2x\{1-(3t/2)^2/2!+(3t/2)^4/4!-\ldots\} - \sin 2x\{3t/2-(3t/2)^3/4!+(3t/2)^5/5!-\ldots\}$$
$$= \cos 2x\cos(3t/2) - \sin 2x\sin(3t/2)$$
$$= \cos[2x+(3t/2)]$$

which is the exact solution.

CONCLUSION

In this paper, we have successfully applied a seemingly New technique, the reduced differential transform method to solve the general linear initial value problem and first order transport equation. The method is applied in a direct way without using linearization, transformation, discretization or restrictive assumptions. The result show RDTM needs small size of computation contrary to other numerical methods (classical differential transform method (DTM), elementary method and method of characteristics) and powerful and efficient techniques in finding the exact solution.

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